21. How do you suppose the first magnets found in Magnesia were formed?

22. Why will either pole of a magnet attract an unmagnetized piece of iron?

23. Suppose you have three iron rods, two of which are magnetized but the third is not. How would you determine which two are the magnets without using any additional objects?

21. Domains in ferromagnetic materials in molten form were aligned by the Earth's magnetic field and then fixed in place as the material cooled.

22. Yes. When a magnet is brought near an unmagnetized piece of iron, the magnet's field causes a temporary alignment of the domains of the iron. If the magnet's north pole is brought near the iron, then the domains align such that the temporary south pole of the iron is facing the magnet, and if the magnet's south pole is closest to the iron, then the alignment will be the opposite. In either case, the magnet and the iron will attract each other.

23. The two rods that have ends that repel each other will be the magnets. The unmagnetized rod will be attracted to both ends of the magnetized rods.

10. (II) A straight stream of protons passes a given point in space at a rate of 2.5×10^9 protons/s. What magnetic field do they produce 2.0 m from the beam?

10. The stream of protons constitutes a current, whose magnitude is found by multiplying the proton rate times the charge of a proton. Then use Eq. 28-1 to calculate the magnetic field.

$$B_{\text{stream}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(2.5\times 10^{9}\ \text{protons/s}\right)\left(1.60\times 10^{-19}\ \text{C/proton}\right)}{2\pi\left(2.0\,\text{m}\right)} = \boxed{4.0\times 10^{-17}\,\text{T}}$$

18. (II) A rectangular loop of wire is placed next to a straight wire, as shown in Fig. 28–37. There is a current of 3.5 A in both wires. Determine the magnitude and direction of the net force on the loop.

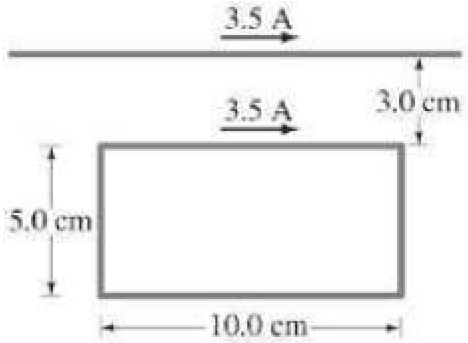


FIGURE 28–37 Problem 18.

18. The magnetic field at the loop due to the long wire is into the page, and can be calculated by Eq. 28-1. The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is more difficult to calculate the total force on the left and right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation. Use Eq. 28-2.

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}}\ell_{\text{near}} - \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}}\ell_{\text{far}} = \frac{\mu_0}{2\pi} I_1 I_2 \ell\left(\frac{1}{d_{\text{near}}} - \frac{1}{d_{\text{far}}}\right)$$

$$= \frac{4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A}}{2\pi}\left(3.5\,\text{A}\right)^2\left(0.100\,\text{m}\right)\left(\frac{1}{0.030\,\text{m}} - \frac{1}{0.080\,\text{m}}\right) = \boxed{5.1\times 10^{-6}\,\text{N, towards wire}}$$

38. (II) A single point charge q is moving with velocity $\vec{\mathbf{v}}$. Use the Biot-Savart law to show that the magnetic field $\vec{\mathbf{B}}$ it produces at a point P, whose position vector relative to the charge is $\vec{\mathbf{r}}$ (Fig. 28–46), is given by

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\vec{\mathbf{r}}}{r^3}.$$

(Assume v is much less than the speed of light.)

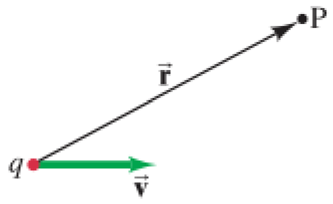
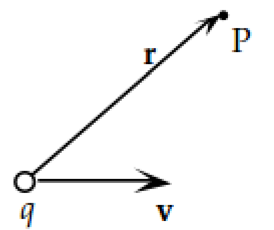


FIGURE 28–46
Problem 38.

38. Treat the moving point charge as a small current segment. We can write the product of the charge and velocity as the product of a current and current segment. Inserting these into the Biot-Savart law gives us the magnetic field at point P.



$$q\vec{\mathbf{v}} = q\frac{d\vec{\ell}}{dt} = \frac{dq}{dt}d\vec{\ell} = I d\vec{\ell}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\ell}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{r}}{r^2} = \boxed{\frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\vec{\mathbf{r}}}{r^3}}$$

47. (II) An iron atom has a magnetic dipole moment of about $1.8\times10^{-23}\,\mathrm{A\cdot m^2}$. (*a*) Determine the dipole moment of an iron bar 9.0 cm long, 1.2 cm wide, and 1.0 cm thick, if it is 100 percent saturated. (*b*) What torque would be exerted on this bar when placed in a 0.80-T field acting at right angles to the bar?

47. (*a*) If the iron bar is completely magnetized, all of the dipoles are aligned. The total dipole moment is equal to the number of atoms times the dipole moment of a single atom.

$$\mu = N\mu_1 = \frac{N_A\rho V}{M_m}\mu_1$$

$$= \frac{(6.022\times10^{23}\,\text{atoms/mole})(7.80\,\text{g/cm}^3)(9.0\,\text{cm})(1.2\,\text{cm})(1.0\,\text{cm})}{55.845\,\text{g/mole}}\left(1.8\times10^{-23}\,\frac{\text{A}\bullet\text{m}^2}{\text{atom}}\right)$$

$$= 16.35\ \text{A}\bullet\text{m}^2 \approx \boxed{16\ \text{A}\bullet\text{m}^2}$$

(*b*) We use Eq. 27-9 to find the torque.

$$\tau = \mu B\sin\theta = (16.35\ \text{A}\bullet\text{m}^2)(0.80\ \text{T})\sin 90^\circ = \boxed{13\ \text{m}\bullet\text{N}}$$

61. Helmholtz coils are two identical circular coils having the same radius R and the same number of turns N, separated by a distance equal to the radius R and carrying the same current I in the same direction. (See Fig. 28–58.) They are used in scientific instruments to generate nearly uniform magnetic fields.(They can be seen in the photo, Fig. 27–18.) (*a*) Determine the magnetic field B at points x along the line joining their centers. Let $x = 0$ at the center of one coil, and $x = R$ at the center of the other. (*b*) Show that the field midway between the coils is particularly uniform by showing that $\frac{dB}{dx} = 0$ and $\frac{d^2B}{dx^2} = 0$ at the midpoint between the coils. (*c*) If $R = 10.0$ cm, $N = 250$ turns and $I = 2.0$ A, what is the field at the midpoint between the coils, $x = R/2$?

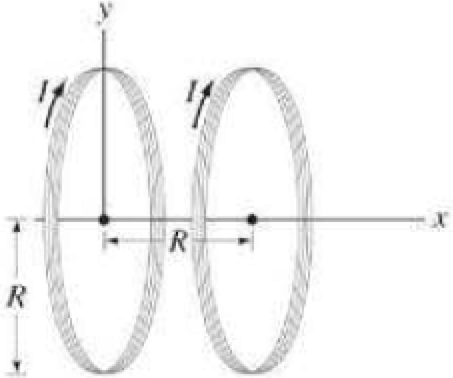


FIGURE 28–58 Problem 61.

61. (*a*) Choose $x = 0$ at the center of one coil. The center of the other coil will then be at $x = R$. Since the currents flow in the same direction in both coils, the right-hand-rule shows that the magnetic fields from the two coils will point in the same direction along the axis. The magnetic field from a current loop was found in Example 28-12. Adding the two magnetic fields together yields the total field.

$$B(x) = \boxed{\frac{\mu_0 NIR^2}{2\left[R^2 + x^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2 + (x-R)^2\right]^{3/2}}}$$

(*b*) Evaluate the derivative of the magnetic field at $x = \frac{1}{2}R$.

$$\frac{dB}{dx} = -\frac{3\mu_0 NIR^2 x}{2\left[R^2 + x^2\right]^{5/2}} - \frac{3\mu_0 NIR^2 (x-R)}{2\left[R^2 + (x-R)^2\right]^{5/2}} = -\frac{3\mu_0 NIR^3}{4\left[R^2 + R^2/4\right]^{5/2}} - \frac{-3\mu_0 NIR^3}{4\left[R^2 + R^2/4\right]^{5/2}} = \boxed{0}$$

Evaluate the second derivative of the magnetic field at $x = \frac{1}{2}R$.

$$\frac{d^2B}{dx^2} = -\frac{3\mu_0 NIR^2}{2\left[R^2 + x^2\right]^{5/2}} + \frac{15\mu_0 NIR^2 x^2}{2\left[R^2 + x^2\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[R^2 + (x-R)^2\right]^{5/2}} + \frac{15\mu_0 NIR^2 (x-R)^2}{2\left[R^2 + (x-R)^2\right]^{7/2}}$$

$$= -\frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}}$$

$$= \frac{\mu_0 NIR^2}{\left[5R^2/4\right]^{5/2}}\left(-\frac{3}{2} + \frac{15\cdot 4}{8\cdot 5} - \frac{3}{2} + \frac{15\cdot 4}{8\cdot 5}\right) = \boxed{0}$$

Therefore, at the midpoint $\frac{dB}{dx} = 0$ and $\frac{d^2B}{dx^2} = 0$.

(*c*) We insert the given data into the magnetic field equation to calculate the field at the midpoint.

$$B\left(\tfrac{1}{2}R\right) = \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}} = \frac{\mu_0 NIR^2}{\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}}$$

$$= \frac{\left(4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A}\right)(250)(2.0\,\text{A})(0.10\,\text{m})^2}{\left[(0.10\,\text{m})^2 + (0.05\,\text{m})^2\right]^{3/2}} = \boxed{4.5\,\text{mT}}$$