7. In Fig. 27–34, charged particles move in the vicinity of a current-carrying wire. For each charged particle, the arrow indicates the direction of motion of the particle, and the + or − indicates the sign of the charge. For each of the particles, indicate the direction of the magnetic force due to the magnetic field produced by the wire.

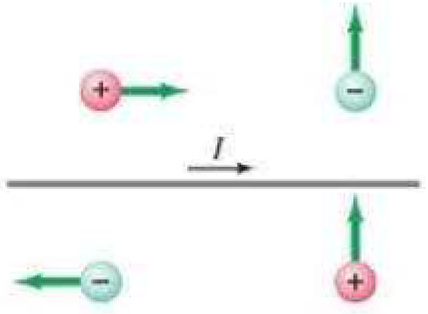


FIGURE 27–34 Question 7.

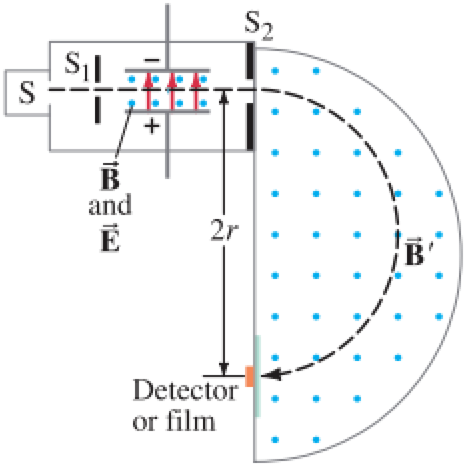
7. Positive particle in the upper left: force is downward toward the wire. Negative particle in the upper right: force is to the left. Positive particle in the lower right: force is to the left. Negative particle in the lower left: force is upward toward the wire.

12. Can you set a resting electron into motion with a steady magnetic field? With an electric field? Explain.

12. No, you cannot set a resting electron into motion with a static magnetic field. In order for a charged particle to experience a magnetic force, it must already have a velocity with a component perpendicular to the magnetic field: $F = qvB\sin\theta$. If $v = 0$, $F = 0$. Yes, you can set an electron into motion with an electric field. The electric force on a charged particle does not depend on velocity: $F = qE$.

***23.** Two ions have the same mass, but one is singly ionized and the other is doubly ionized. How will their positions on the film of the mass spectrometer of Fig. 27–33 differ?

FIGURE 27–33 Bainbridge-type mass spectrometer. The magnetic fields B and B' point out of the paper (indicated by the dots), for positive ions.



23. The distance $2r$ to the singly charged ions will be twice the distance to the doubly charged ions.

8. (II) A long wire stretches along the x axis and carries a 3.0-A current to the right $(+x)$. The wire is in a uniform magnetic field $\vec{\mathbf{B}} = (0.20\hat{\mathbf{i}} - 0.36\hat{\mathbf{j}} + 0.25\hat{\mathbf{k}})\,\mathrm{T}$. Determine the components of the force on the wire per cm of length.

8. We find the force per unit length from Eq. 27-3. Note that while the length is not known, the direction is given, and so $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}}_B = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = I\ell\hat{\mathbf{i}} \times \vec{\mathbf{B}} \rightarrow$$

$$\frac{\vec{\mathbf{F}}_B}{\ell} = I\hat{\mathbf{i}} \times \vec{\mathbf{B}} = (3.0\,\mathrm{A})\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ 0.20\,\mathrm{T} & -0.36\,\mathrm{T} & 0.25\,\mathrm{T} \end{vmatrix} = (-0.75\hat{\mathbf{j}} - 1.08\hat{\mathbf{k}})\,\mathrm{N/m}\left(\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)$$

$$= \boxed{-(7.5\hat{\mathbf{j}} + 11\hat{\mathbf{k}}) \times 10^{-3}\,\mathrm{N/cm}}$$

14. (I) An electron is projected vertically upward with a speed of $1.70 \times 10^6\,\mathrm{m/s}$ into a uniform magnetic field of 0.480 T that is directed horizontally away from the observer. Describe the electron's path in this field.

14. The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

$$F_{\max} = qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31}\,\mathrm{kg})(1.70 \times 10^6\,\mathrm{m/s})}{(1.60 \times 10^{-19}\,\mathrm{C})(0.480\,\mathrm{T})} = \boxed{2.02 \times 10^{-5}\,\mathrm{m}}$$

16. (I) Find the direction of the force on a negative charge for each diagram shown in Fig. 27–42, where $\vec{\mathbf{v}}$ (green) is the velocity of the charge and $\vec{\mathbf{B}}$ (blue) is the direction of the magnetic field. (⊗ means the vector points inward. ⊙ means it points outward, toward you.)

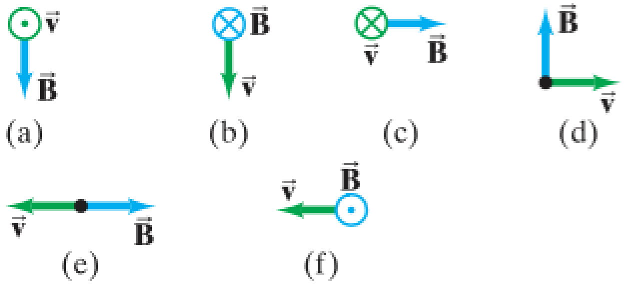


FIGURE 27–42 Problem 16.

16. Since the charge is negative, the answer is the OPPOSITE of the result given from the right hand rule applied to the velocity and magnetic field.

(*a*) left
(*b*) left
(*c*) upward
(*d*) inward into the paper
(*e*) no force
(*f*) downward

19. (II) A doubly charged helium atom whose mass is 6.6×10^{-27} kg is accelerated by a voltage of 2700 V. (*a*) What will be its radius of curvature if it moves in a plane perpendicular to a uniform 0.340-T field? (*b*) What is its period of revolution?

19. (*a*) The velocity of the ion can be found using energy conservation. The electrical potential energy of the ion becomes kinetic energy as it is accelerated. Then, since the ion is moving perpendicular to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow qV = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$F_{\max} = qvB = m\frac{v^2}{r} \rightarrow$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{0.340\,\text{T}}\sqrt{\frac{2(6.6\times10^{-27}\,\text{kg})(2700\,\text{V})}{2(1.60\times10^{-19}\,\text{C})}} = \boxed{3.1\times10^{-2}\,\text{m}}$$

(*b*) The period can be found from the speed and the radius. Use the expressions for the radius and the speed from above.

$$v = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{v} = \frac{2\pi\frac{1}{B}\sqrt{\frac{2mV}{q}}}{\sqrt{\frac{2qV}{m}}} = \frac{2\pi m}{qB} = \frac{2\pi(6.6\times10^{-27}\,\text{kg})}{2(1.60\times10^{-19}\,\text{C})(0.340\,\text{T})} = \boxed{3.8\times10^{-7}\,\text{s}}$$

27. (II) A particle of charge q moves in a circular path of radius r in a uniform magnetic field $\vec{\mathbf{B}}$. If the magnitude of the magnetic field is doubled, and the kinetic energy of the particle remains constant, what happens to the angular momentum of the particle?

27. The kinetic energy of the particle can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined. Inserting the radius and velocity into the equation for angular momentum gives the angular momentum in terms of the kinetic energy and magnetic field.

$$K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB}$$

$$L = mvr = m\sqrt{\frac{2K}{m}}\left(\frac{m\sqrt{\frac{2K}{m}}}{qB}\right) = \frac{2mK}{qB}$$

From the equation for the angular momentum, we see that doubling the magnetic field while keeping the kinetic energy constant will cut the angular momentum in half.

$$\boxed{L_{\text{final}} = \tfrac{1}{2}L_{\text{initial}}}$$

38. (II) Show that the magnetic dipole moment μ of an electron orbiting the proton nucleus of a hydrogen atom is related to the orbital momentum L of the electron by

$$\mu = \frac{e}{2m}L.$$

38. The magnetic dipole moment is defined in Eq. 27-10 as $\mu = NIA$. The number of turns, N, is 1. The current is the charge per unit time passing a given point, which on the average is the charge on the electron divided by the period of the circular motion, $I = e/T$. If we assume the electron is moving in a circular orbit of radius r, then the area is πr^2. The period of the motion is the circumference of the orbit divided by the speed, $T = 2\pi r/v$. Finally, the angular momentum of an object moving in a circle is given by $L = mrv$. Combine these relationships to find the magnetic moment.

$$\mu = NIA = \frac{e}{T}\pi r^2 = \frac{e}{2\pi r/v}\pi r^2 = \frac{e\pi r^2 v}{2\pi r} = \frac{erv}{2} = \frac{emrv}{2m} = \frac{e}{2m}mrv = \frac{e}{2m}L$$

66. The **cyclotron** (Fig. 27–50) is a device used to accelerate elementary particles such as protons to high speeds. Particles starting at point A with some initial velocity travel in circular orbits in the magnetic field B. The particles are accelerated to higher speeds each time they pass in the gap between the metal "dees," where there is an electric field E. (There is no electric field within the hollow metal dees.) The electric field changes direction each half-cycle, due to an ac voltage $V = V_0 \sin 2\pi f t$, so that the particles are increased in speed at each passage through the gap. (*a*) Show that the frequency f of the voltage must be $f = Bq/2\pi m$, where q is the charge on the particles and m their mass. (*b*) Show that the kinetic energy of the particles increases by $2qV_0$ each revolution, assuming that the gap is small. (*c*) If the radius of the cyclotron is 0.50 m and the magnetic field strength is 0.60 T, what will be the maximum kinetic energy of accelerated protons in MeV?

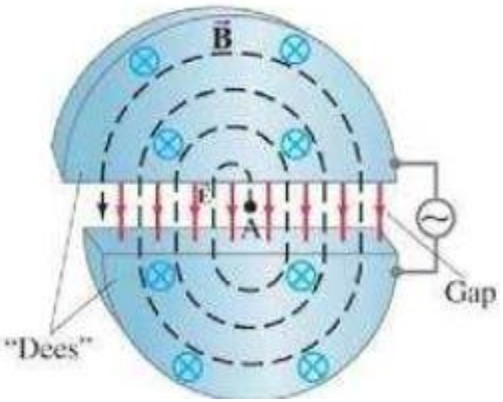


FIGURE 27–50 A cyclotron. Problem 66

66. (*a*) The frequency of the voltage must match the frequency of circular motion of the particles, so that the electric field is synchronized with the circular motion. The radius of each circular orbit is given in Example 27-7 as $r = \dfrac{mv}{qB}$. For an object moving in circular motion, the period is given by $T = \dfrac{2\pi r}{v}$, and the frequency is the reciprocal of the period.

$$T = \frac{2\pi r}{v} \rightarrow f = \frac{v}{2\pi r} = \frac{v}{2\pi \frac{mv}{qB}} = \boxed{\frac{Bq}{2\pi m}}$$

In particular we note that this frequency is independent of the radius, and so the same frequency can be used throughout the acceleration.

(*b*) For a small gap, the electric field across the gap will be approximately constant and uniform as the particles cross the gap. If the motion and the voltage are synchronized so that the maximum voltage occurs when the particles are at the gap, the particles receive an energy increase of $K = qV_0$ as they pass each gap. The energy gain from one revolution will include the passing of 2 gaps, so the total kinetic energy increase is $\boxed{2qV_0}$.

(*c*) The maximum kinetic energy will occur at the outside of the cyclotron.

$$K_{max} = \tfrac{1}{2} m v_{max}^2 = \tfrac{1}{2} m \left(\frac{r_{max} qB}{m}\right)^2 = \tfrac{1}{2} \frac{r_{max}^2 q^2 B^2}{m} = \tfrac{1}{2} \frac{(0.50\,\text{m})^2 (1.60\times10^{-19}\,\text{C})^2 (0.60\,\text{T})^2}{1.67\times10^{-27}\,\text{kg}}$$

$$= 6.898\times10^{-13}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{10^6\,\text{eV}}\right) = \boxed{4.3\,\text{MeV}}$$

67. Magnetic fields are very useful in particle accelerators for "beam steering"; that is, magnetic fields can be used to change the beam's direction without altering its speed (Fig. 27–51). Show how this could work with a beam of protons. What happens to protons that are not moving with the speed that the magnetic field is designed for? If the field extends over a region 5.0 cm wide and has a magnitude of 0.38 T, by approximately what angle will a beam of protons traveling at 0.85×10^7 m/s be bent?

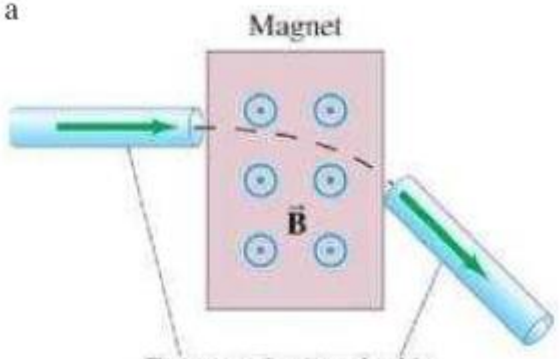
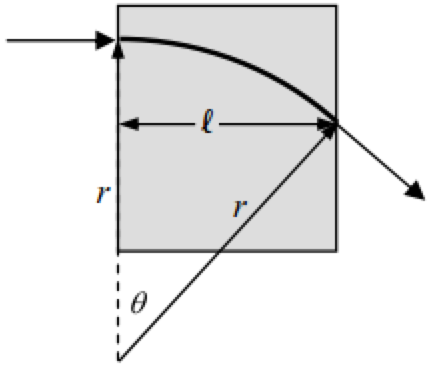


FIGURE 27–51 Problem 67.

67. The protons will follow a circular path as they move through the region of magnetic field, with a radius of curvature given in Example 27-7 as $r = \frac{mv}{qB}$. Fast-moving protons will have a radius of curvature that is too large and so they will exit above the second tube. Likewise, slow-moving protons will have a radius of curvature that is too small and so they will exit below the second tube. Since the exit velocity is perpendicular to the radius line from the center of curvature, the bending angle can be calculated.



$$\sin\theta = \frac{\ell}{r} \quad \rightarrow$$

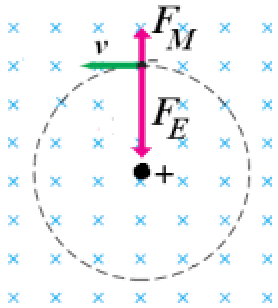
$$\theta = \sin^{-1}\frac{\ell}{r} = \sin^{-1}\frac{\ell qB}{mv} = \sin^{-1}\frac{(5.0\times10^{-2}\,\text{m})(1.60\times10^{-19}\,\text{C})(0.38\,\text{T})}{(1.67\times10^{-27}\,\text{kg})(0.85\times10^{7}\,\text{m/s})} = \sin^{-1}0.214 = \boxed{12^\circ}$$

72. Zeeman effect. In the Bohr model of the hydrogen atom, the electron is held in its circular orbit of radius r about its proton nucleus by electrostatic attraction. If the atoms are placed in a weak magnetic field $\vec{\mathbf{B}}$, the rotation frequency of electrons rotating in a plane perpendicular to $\vec{\mathbf{B}}$ is changed by an amount

$$\Delta f = \pm \frac{eB}{4\pi m}$$

where e and m are the charge and mass of an electron. (*a*) Derive this result, assuming the force due to $\vec{\mathbf{B}}$ is much less than that due to electrostatic attraction of the nucleus. (*b*) What does the $\pm$ sign indicate?

72. (*a*) As the electron orbits the nucleus in the absence of the magnetic field, its centripetal acceleration is caused solely by the electrical attraction between the electron and the nucleus. Writing the velocity of the electron as the circumference of its orbit times its frequency, enables us to obtain an equation for the frequency of the electron's orbit.



$$\frac{ke^2}{r^2} = m\frac{v^2}{r} = m\frac{(2\pi r f_0)^2}{r} \rightarrow f_0^2 = \frac{ke^2}{4\pi^2 m r^3}$$

When the magnetic field is added, the magnetic force adds or subtracts from the centripetal acceleration (depending on the direction of the field) resulting in the change in frequency.

$$\frac{ke^2}{r^2} \pm q(2\pi r f)B = m\frac{(2\pi r f)^2}{r} \rightarrow f^2 \mp \frac{qB}{2\pi m} f - f_0^2 = 0$$

We can solve for the frequency shift by setting $f = f_0 + \Delta f$, and only keeping the lowest order terms, since $\Delta f \ll f_0$.

$$(f_0 + \Delta f)^2 \mp \frac{qB}{2\pi m}(f_0 + \Delta f) - f_0^2 = 0$$

$$\cancel{f_0^2} + 2f_0\Delta f + \cancel{\Delta f^2} \mp \frac{qB}{2\pi m} f_0 \mp \cancel{\frac{qB}{2\pi m}\Delta f} - \cancel{f_0^2} = 0 \rightarrow \boxed{\Delta f = \pm\frac{qB}{4\pi m}}$$

(*b*) The "±" indicates whether the magnetic force adds to or subtracts from the centripetal acceleration. If the magnetic force adds to the centripetal acceleration, the frequency increases. If the magnetic force is opposite in direction to the acceleration, the frequency decreases.