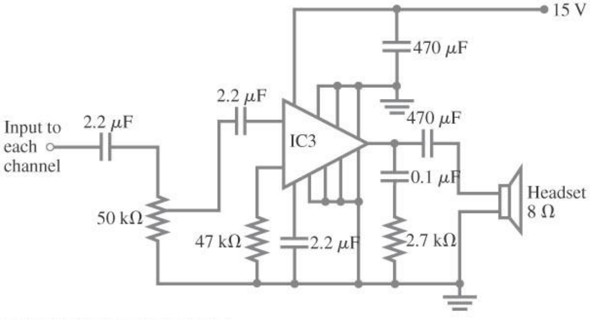
Κεφάλαιο 26
DC Circuits-Συνεχή Ρεύματα



Περιεχόμενα Κεφαλαίου 26

- **Ηλεκτρεγερτική Δύναμη (ΗΕΔ)**
- **Αντιστάσεις σε σειρά και Παράλληλες**
- **Νόμοι του Kirchhoff**
- **Κυκλώματα σε Σειρά και Παράλληλα**
- **EMF-Φόρτιση Μπαταρίας**
- **Κυκλώματα *RC***
- **Μέτρηση Τάσεως και Ρεύματος**

26-1 ΗΕΔ

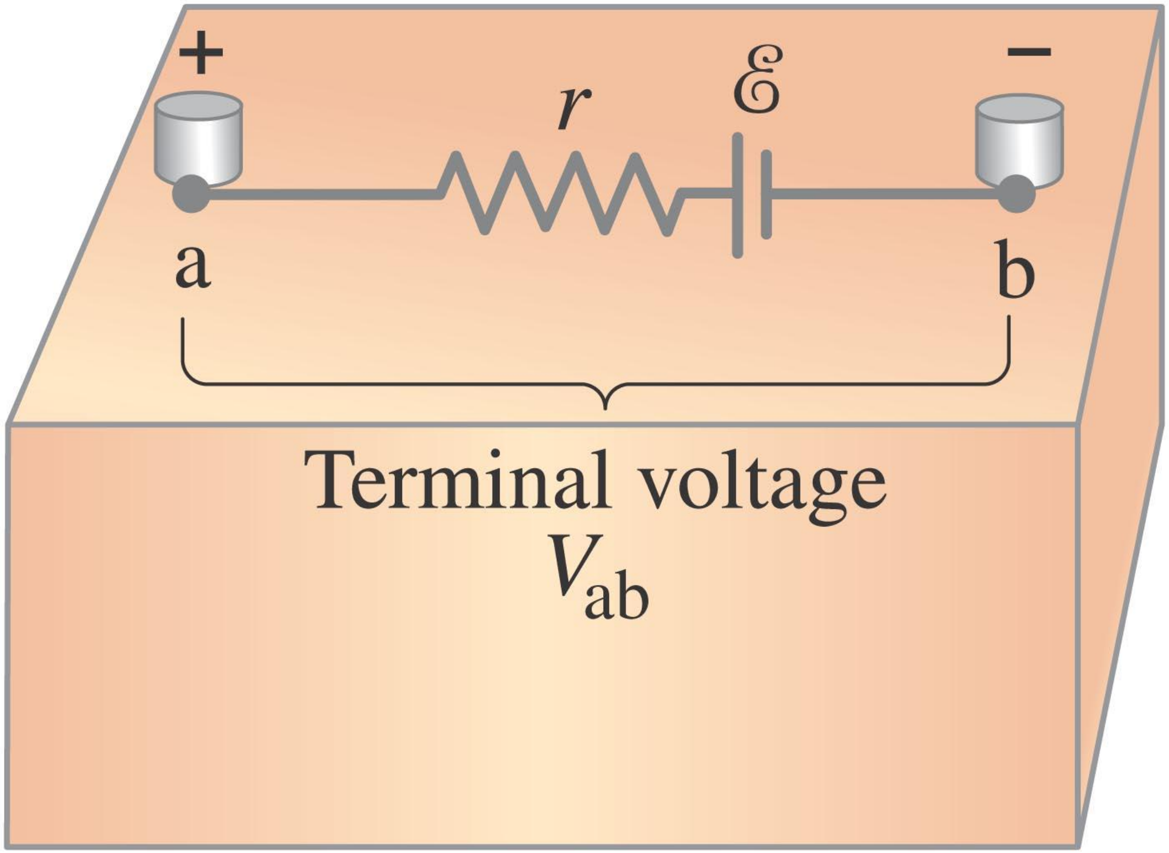
Ένα ηλεκτρικό κύκλωμα απαιτεί κάποια μπαταρία ή δυναμό προκειμένου να «παράγει» ρεύμα. Οι πηγές αυτές ονομάζονται πηγές ηλεκτρεγερτικής δύναμης (ΗΕΔ).

Η μπαταρία είναι πηγή σταθερής τάσεως αλλά έχει μια μικρή εσωτερική αντίσταση η οποία μειώνει την τάση που αποδίδει από αυτή μιας ιδανικής πηγής ΗΕΔ:

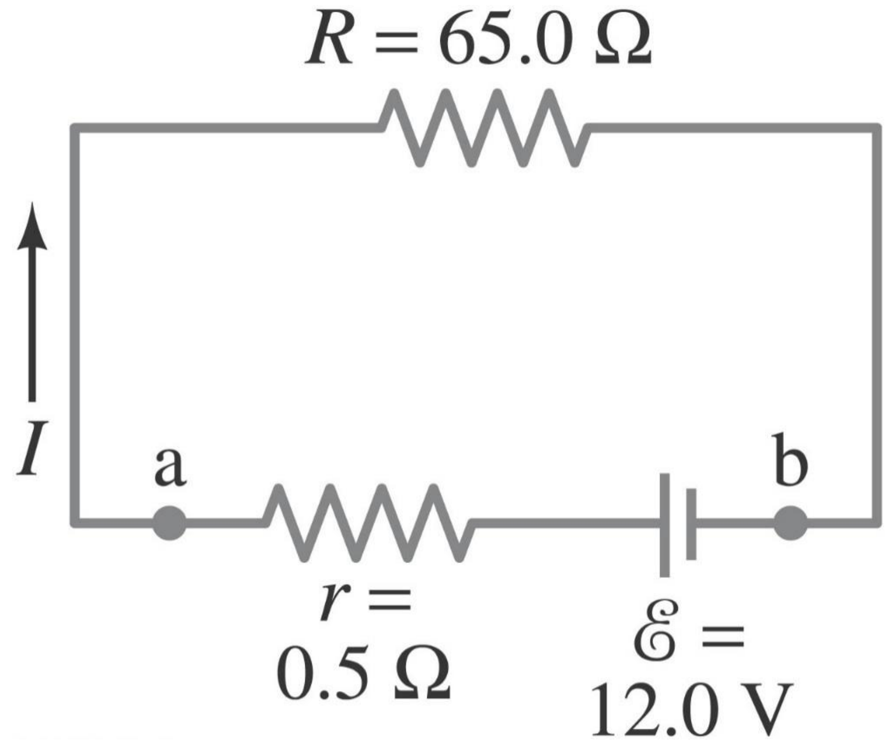
$$V_{\text{ab}} = \mathscr{E} - Ir.$$

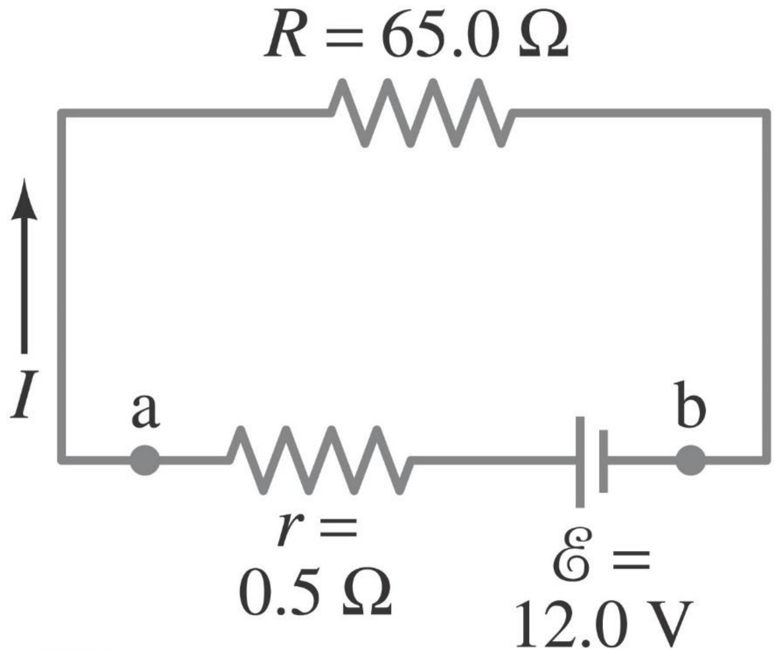
26-1 ΗΕΔ και Τάση στου Πόλους

Η εσωτερική αυτή αντίσταση συμπεριφέρεται ως να ήταν σε σειρά με την ΗΕΔ.



Μια 65.0-Ω αντίσταση συνδέεται με μπαταρία 12.0 V που έχει εσωτερική αντίσταση 0.5 Ω. Βρείτε (a) το ρεύμα του κυκλώματος (b) την τάση εξόδου της μπαταρίας V_{ab}, και (c) την ισχύ που καταναλώνεται στις αντιστάσεις R και r.





SOLUTION (a) From Eq. 26–1, we have

$$V_{\mathrm{ab}} = \mathscr{E} - Ir.$$

We apply Ohm's law (Eqs. 25–2) to this battery and the resistance R of the circuit: $V_{\mathrm{ab}} = IR$. Hence $IR = \mathscr{E} - Ir$ or $\mathscr{E} = I(R + r)$, and so

$$I = \frac{\mathscr{E}}{R + r} = \frac{12.0\ \mathrm{V}}{65.0\ \Omega + 0.5\ \Omega} = \frac{12.0\ \mathrm{V}}{65.5\ \Omega} = 0.183\ \mathrm{A}.$$

(b) The terminal voltage is

$$V_{\mathrm{ab}} = \mathscr{E} - Ir = 12.0\ \mathrm{V} - (0.183\ \mathrm{A})(0.5\ \Omega) = 11.9\ \mathrm{V}.$$

(c) The power dissipated (Eq. 25–7) in R is

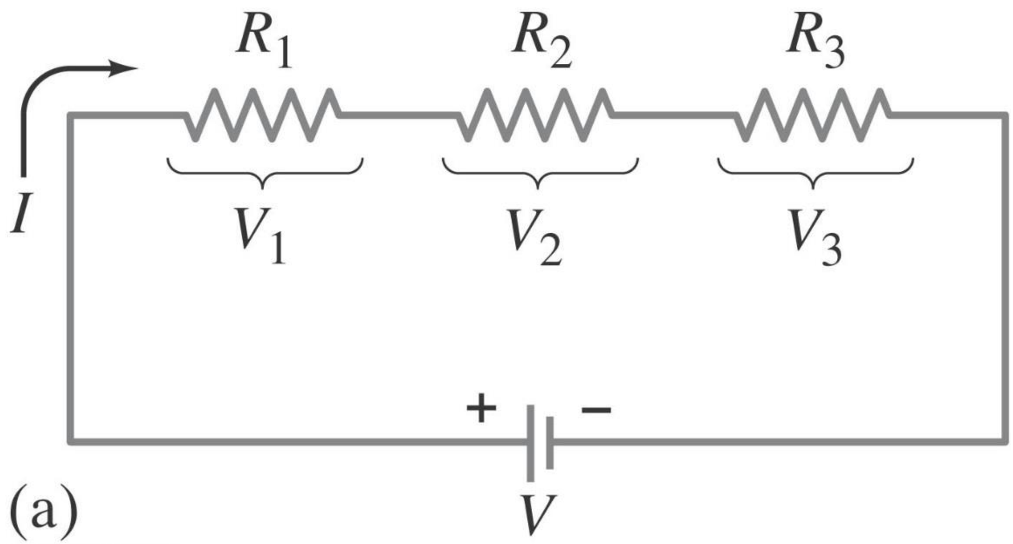
$$P_R = I^2R = (0.183\ \mathrm{A})^2(65.0\ \Omega) = 2.18\ \mathrm{W},$$

and in r is

$$P_r = I^2r = (0.183\ \mathrm{A})^2(0.5\ \Omega) = 0.02\ \mathrm{W}.$$

26-2 Αντιστάσεις σε Σειρά

Σύνδεση σε σειρά: <u>**έχουμε κοινό ρεύμα**</u>



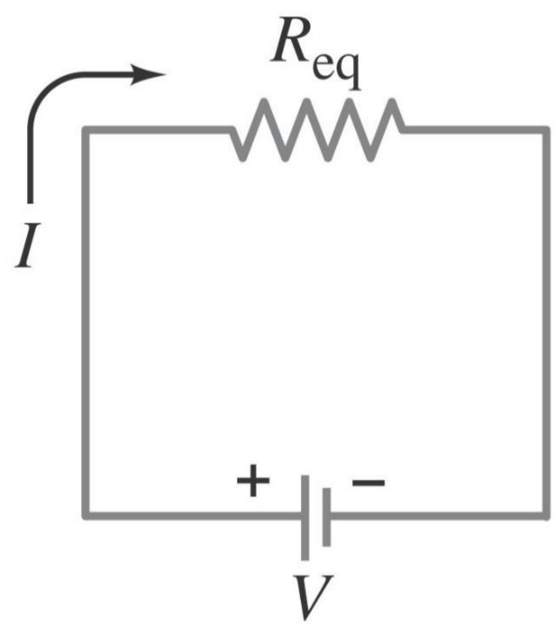
Η πτώση τάσεως είναι ανάλογη της αντίστασης

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3. \qquad \text{[series]}$$

26-2 Αντιστάσεις σε Σειρά

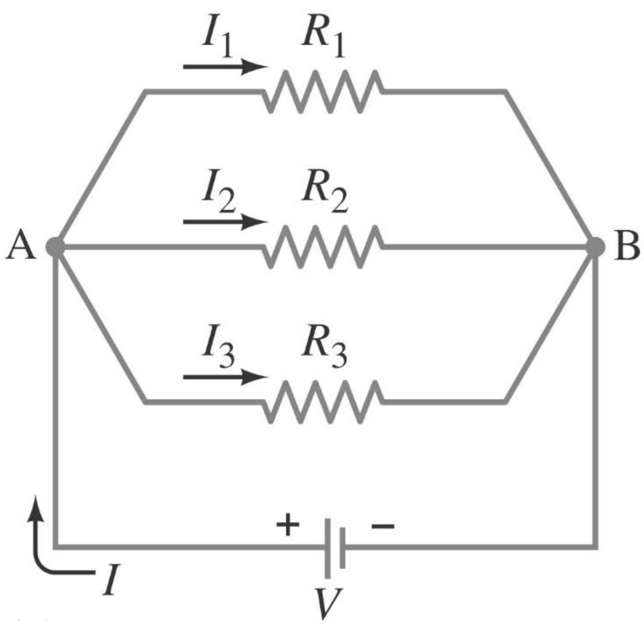
Η συνολική αντίσταση είναι το άθροισμα των αντιστάσεων

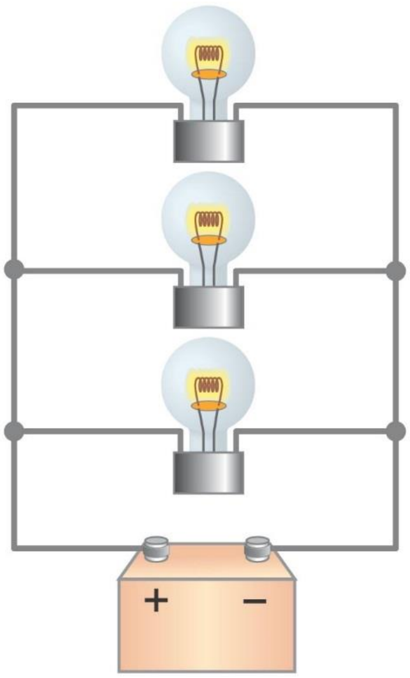
$$R_{eq} = R_1 + R_2 + R_3. \qquad \text{[series]}$$

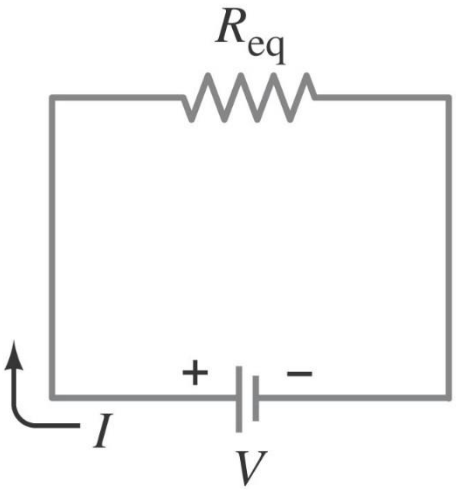


26-2 Παράλληλες Αντιστάσεις

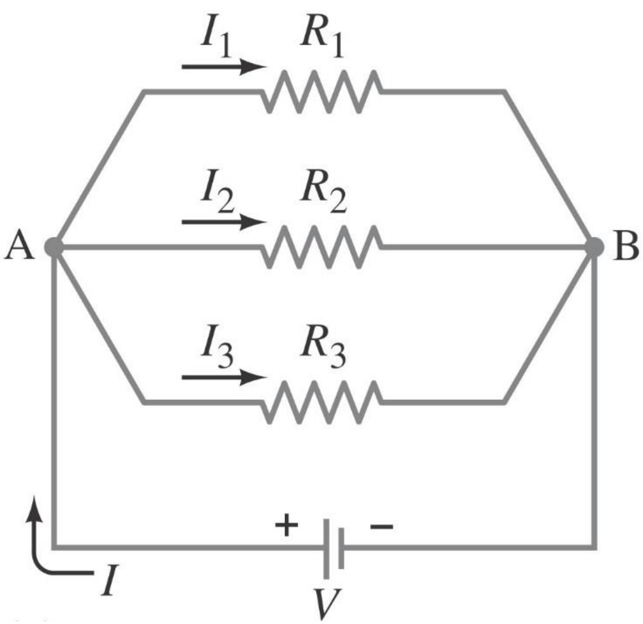
Στην παράλληλη σύνδεση: <u>**έχουμε ίδια Τάση**</u>







26-2 Παράλληλες Αντιστάσεις

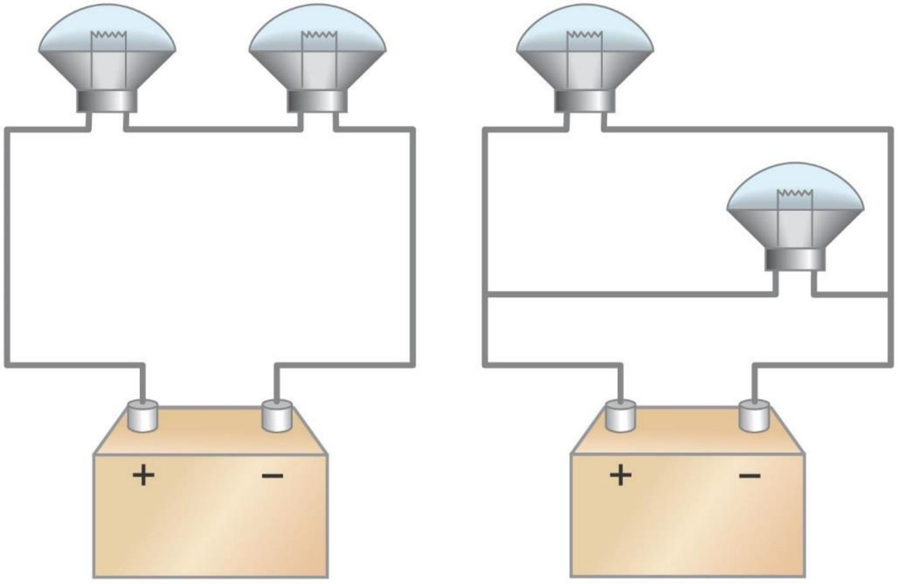


$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

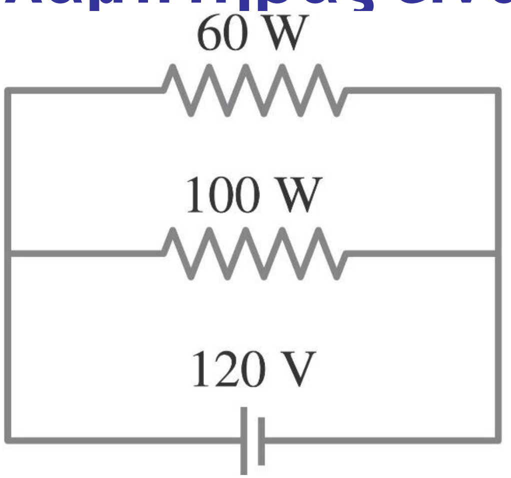
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \qquad \text{[parallel]}$$

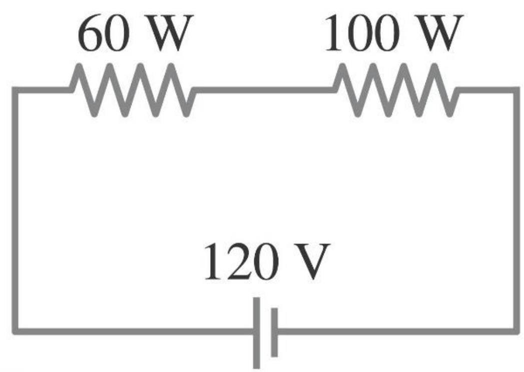
(a) Τα λαμπάκια του παραδείγματος είναι πανομοιότυπα. Σε ποια σύνδεση έχουμε μεγαλύτερη φωτεινότητα (b) Με ποιον από τους δύο τρόπους συνδέονται τα φώτα του αυτοκινήτου;



Η παράλληλη σύνδεση είναι η σωστή!

Δύο λαμπτήρες, 100-W, 120-V και 60-W, 120-V συνδέονται με δύο διαφορετικούς τρόπους. Αγνοώντας την μεταβολή της αντίσταση λόγο θερμότητας, βρείτε ποιος λαμπτήρας είναι φωτεινότερος



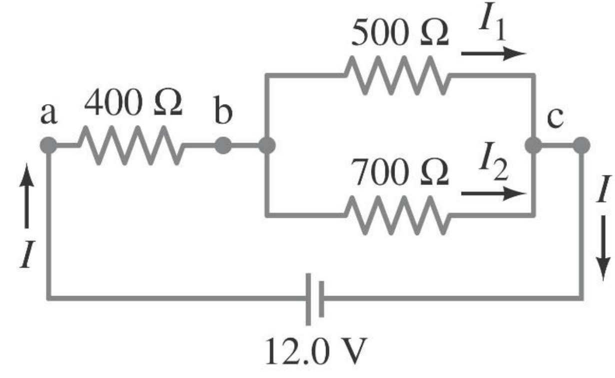


RESPONSE (*a*) These are normal lightbulbs with their power rating given for 120 V. They both receive 120 V, so the 100-W bulb is naturally brighter.

(*b*) The resistance of the 100-W bulb is less than that of the 60-W bulb (calculated from $P = V^2/R$ at constant 120 V). Here they are connected in series and receive the same current. Hence, from $P = I^2R$ (I = constant) the higher-resistance "60-W" bulb will transform more power and thus be brighter.

NOTE When connected in series as in (b), the two bulbs do *not* dissipate 60 W and 100 W because neither bulb receives 120 V.

Πόσο ρεύμα τραβάει το κύκλωμα από τη μπαταρία;



SOLUTION The equivalent resistance, R_P, of the 500-Ω and 700-Ω resistors in parallel is given by

$$\frac{1}{R_P} = \frac{1}{500\,\Omega} + \frac{1}{700\,\Omega} = 0.0020\,\Omega^{-1} + 0.0014\,\Omega^{-1} = 0.0034\,\Omega^{-1}.$$

This is $1/R_P$, so we take the reciprocal to find R_P. It is a common mistake to forget to take this reciprocal. Notice that the units of reciprocal ohms, Ω^{-1}, are a reminder. Thus

$$R_P = \frac{1}{0.0034\,\Omega^{-1}} = 290\,\Omega.$$

This 290 Ω is the equivalent resistance of the two parallel resistors, and is in series with the 400-Ω resistor as shown in the equivalent circuit of Fig. 26–8b. To find the total equivalent resistance R_{eq}, we add the 400-Ω and 290-Ω resistances together, since they are in series, and find

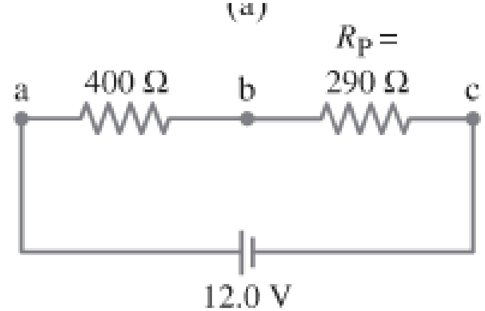
$$R_{eq} = 400\,\Omega + 290\,\Omega = 690\,\Omega.$$

The total current flowing from the battery is then

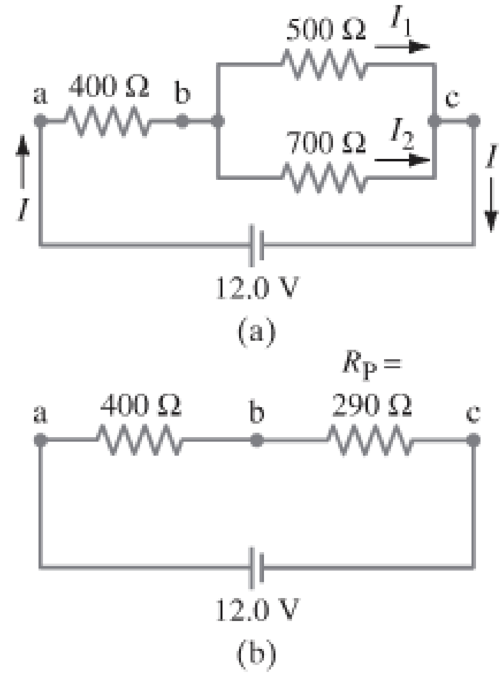
$$I = \frac{V}{R_{eq}} = \frac{12.0\,\text{V}}{690\,\Omega} = 0.0174\,\text{A} \approx 17\,\text{mA}.$$

NOTE This I is also the current flowing through the 400-Ω resistor, but not through the 500-Ω and 700-Ω resistors (both currents are less—see the next Example).

NOTE Complex resistor circuits can often be analyzed in this way, considering the circuit as a combination of series and parallel resistances.



Πόσο είναι το ρεύμα I_1;



APPROACH We need to find the voltage across the 500-Ω resistor, which is the voltage between points b and c in Fig. 26–8a, and we call it V_{bc}. Once V_{bc} is known, we can apply Ohm's law, $V = IR$, to get the current. First we find the voltage across the 400-Ω resistor, V_{ab}, since we know that 17.4 mA passes through it (Example 26–4).

SOLUTION V_{ab} can be found using $V = IR$:

$$V_{ab} = (0.0174\ \text{A})(400\ \Omega) = 7.0\ \text{V}.$$

Since the total voltage across the network of resistors is $V_{ac} = 12.0\ \text{V}$, then V_{bc} must be $12.0\ \text{V} - 7.0\ \text{V} = 5.0\ \text{V}$. Then Ohm's law applied to the 500-Ω resistor tells us that the current I_1 through that resistor is

$$I_1 = \frac{5.0\ \text{V}}{500\ \Omega} = 1.0 \times 10^{-2}\ \text{A} = 10\ \text{mA}.$$

This is the answer we wanted. We can also calculate the current I_2 through the 700-Ω resistor since the voltage across it is also 5.0 V:

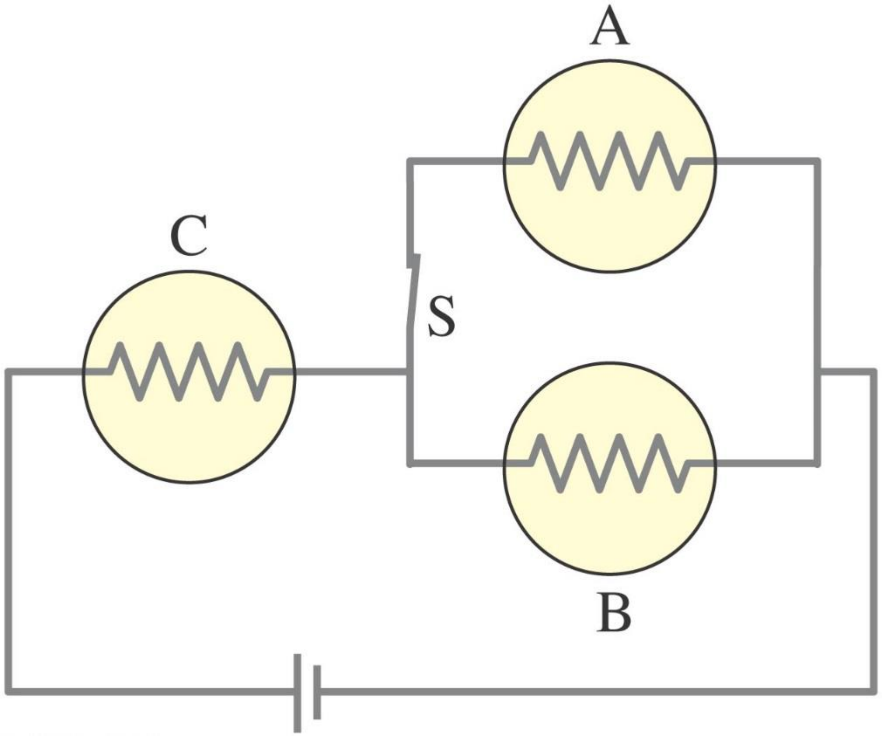
$$I_2 = \frac{5.0\ \text{V}}{700\ \Omega} = 7\ \text{mA}.$$

NOTE When I_1 combines with I_2 to form the total current I (at point c in Fig. 26–8a), their sum is $10\ \text{mA} + 7\ \text{mA} = 17\ \text{mA}$. This equals the total current I as calculated in Example 26–4, as it should.

Θεωρούμε τρείς ίδιους λαμπτήρες με αντίσταση R.

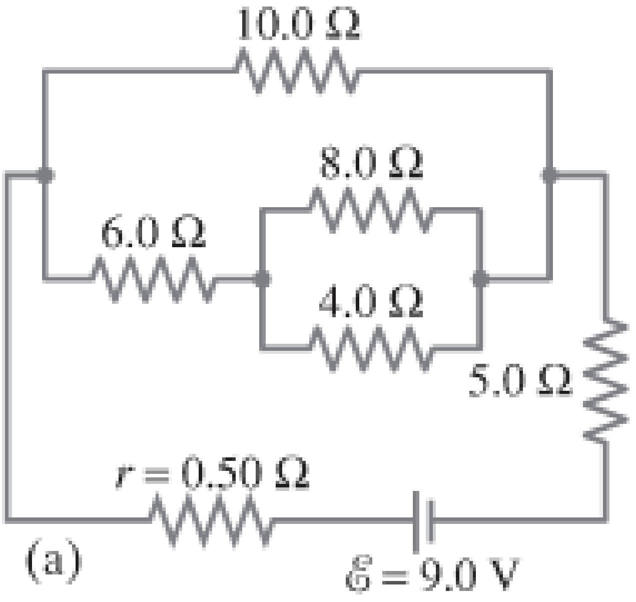
(a) Με τον διακόπτη S «κλειστό» πως συγκρίνει η φωτεινότητα των A και B με αυτήν της C;

(b) Τι συμβαίνει όταν «ανοίξει» ο διακόπτης;

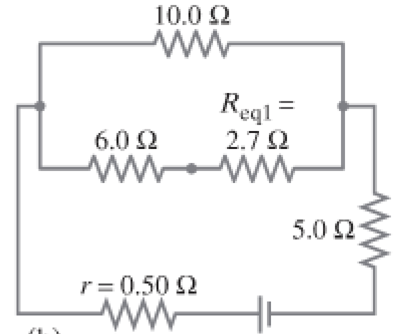


RESPONSE (*a*) With switch S closed, the current that passes through bulb C must split into two equal parts when it reaches the junction leading to bulbs A and B. It splits into equal parts because the resistance of bulb A equals that of B. Thus, bulbs A and B each receive half of C's current; A and B will be equally bright, but they will be less bright than bulb C ($P = I^2R$). (*b*) When the switch S is open, no current can flow through bulb A, so it will be dark. We now have a simple one-loop series circuit, and we expect bulbs B and C to be equally bright. However, the equivalent resistance of this circuit ($= R + R$) is greater than that of the circuit with the switch closed. When we open the switch, we increase the resistance and reduce the current leaving the battery. Thus, bulb C will be dimmer when we open the switch. Bulb B gets more current when the switch is open (you may have to use some mathematics here), and so it will be brighter than with the switch closed; and B will be as bright as C.

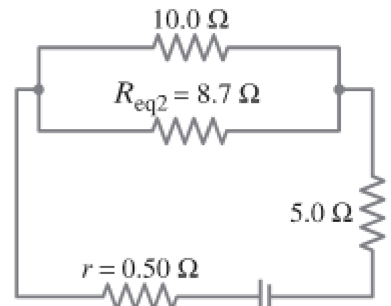
Για το κύκλωμα του σχήματος βρείτε: (a) Το ρεύμα που τραβάμε από την μπαταρία (b) Την τάση εξόδου της μπαταρίας (c) Το ρεύμα μέσα από την αντίσταση των 6.0-Ω.



APPROACH To find the current out of the battery, we first need to determine the equivalent resistance R_{eq} of the entire circuit, including r, which we do by identifying and isolating simple series or parallel combinations of resistors. Once we find I from Ohm's law, $I = \mathscr{E}/R_{eq}$, we get the terminal voltage using $V_{ab} = \mathscr{E} - Ir$. For ($c$) we apply Ohm's law to the 6.0-Ω resistor.



(b)



(c)

SOLUTION (a) We want to determine the equivalent resistance of the circuit. But where do we start? We note that the 4.0-Ω and 8.0-Ω resistors are in parallel, and so have an equivalent resistance R_{eq1} given by

$$\frac{1}{R_{eq1}} = \frac{1}{8.0\,\Omega} + \frac{1}{4.0\,\Omega} = \frac{3}{8.0\,\Omega};$$

so $R_{eq1} = 2.7\,\Omega$. This $2.7\,\Omega$ is in series with the 6.0-Ω resistor, as shown in the equivalent circuit of Fig. 26–10b. The net resistance of the lower arm of the circuit is then

$$R_{eq2} = 6.0\,\Omega + 2.7\,\Omega = 8.7\,\Omega,$$

as shown in Fig. 26–10c. The equivalent resistance R_{eq3} of the 8.7-Ω and 10.0-Ω resistances in parallel is given by

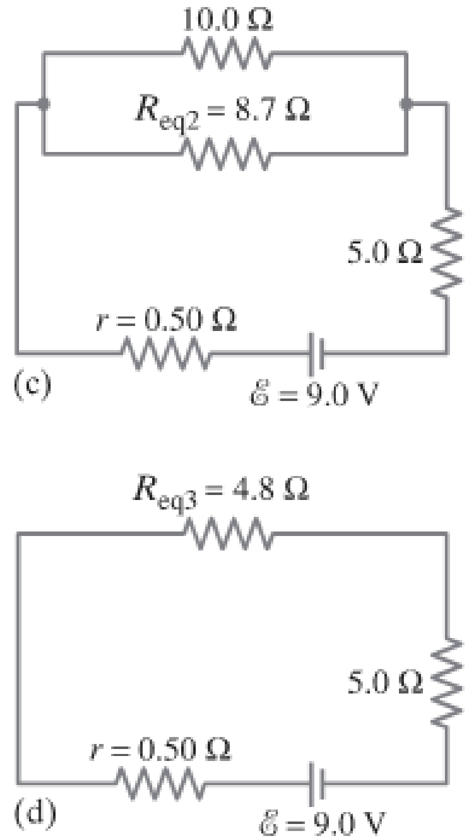
$$\frac{1}{R_{eq3}} = \frac{1}{10.0\,\Omega} + \frac{1}{8.7\,\Omega} = 0.21\,\Omega^{-1},$$

so $R_{eq3} = \left(1/0.21\,\Omega^{-1}\right) = 4.8\,\Omega$. This $4.8\,\Omega$ is in series with the 5.0-Ω resistor and the 0.50-Ω internal resistance of the battery (Fig. 26–10d), so the total equivalent resistance R_{eq} of the circuit is $R_{eq} = 4.8\,\Omega + 5.0\,\Omega + 0.50\,\Omega = 10.3\,\Omega$. Hence the current drawn is

$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{9.0\,\text{V}}{10.3\,\Omega} = 0.87\,\text{A}.$$

(b) The terminal voltage of the battery is

$$V_{ab} = \mathscr{E} - Ir = 9.0\,\text{V} - (0.87\,\text{A})(0.50\,\Omega) = 8.6\,\text{V}.$$



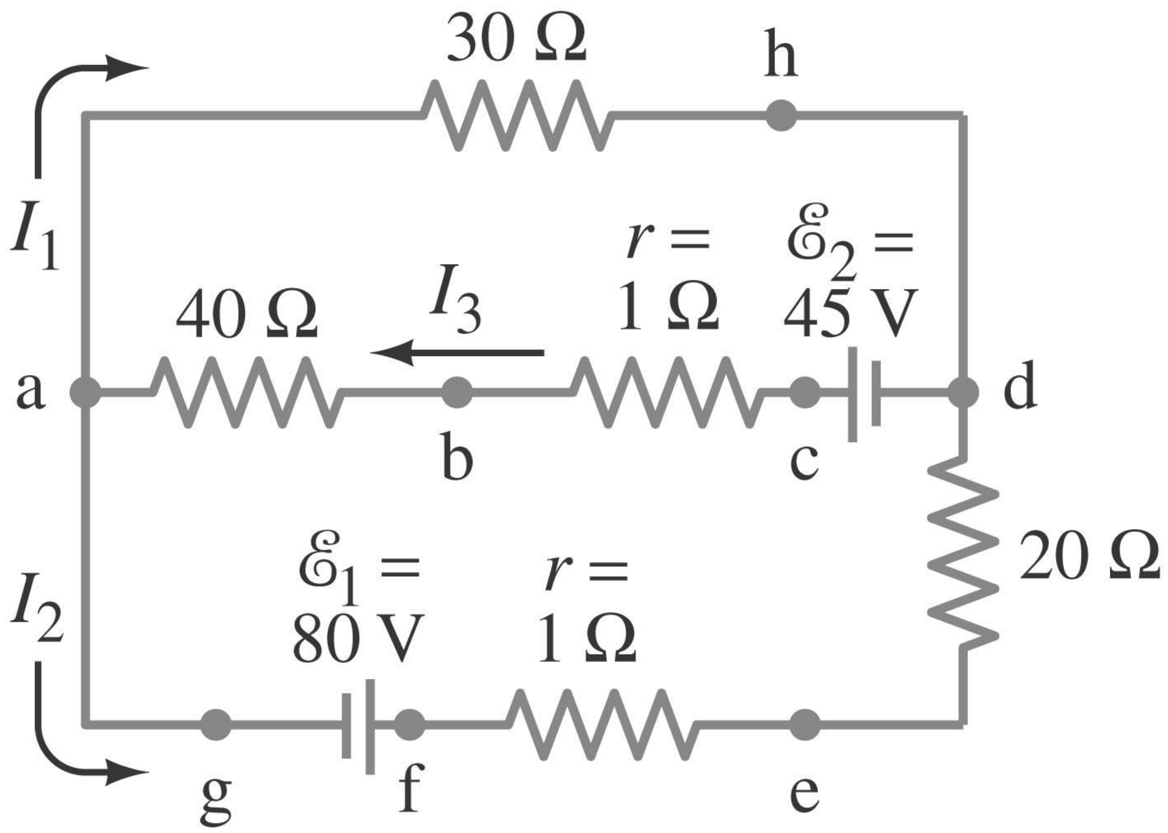
(*c*) Now we can work back and get the current in the 6.0-Ω resistor. It must be the same as the current through the 8.7 Ω shown in Fig. 26–10c (why?). The voltage across that 8.7 Ω will be the emf of the battery minus the voltage drops across r and the 5.0-Ω resistor: $V_{8.7} = 9.0\text{ V} - (0.87\text{ A})(0.50\,\Omega + 5.0\,\Omega)$. Applying Ohm's law, we get the current (call it I')

$$I' = \frac{9.0\text{ V} - (0.87\text{ A})(0.50\,\Omega + 5.0\,\Omega)}{8.7\,\Omega} = 0.48\text{ A}.$$

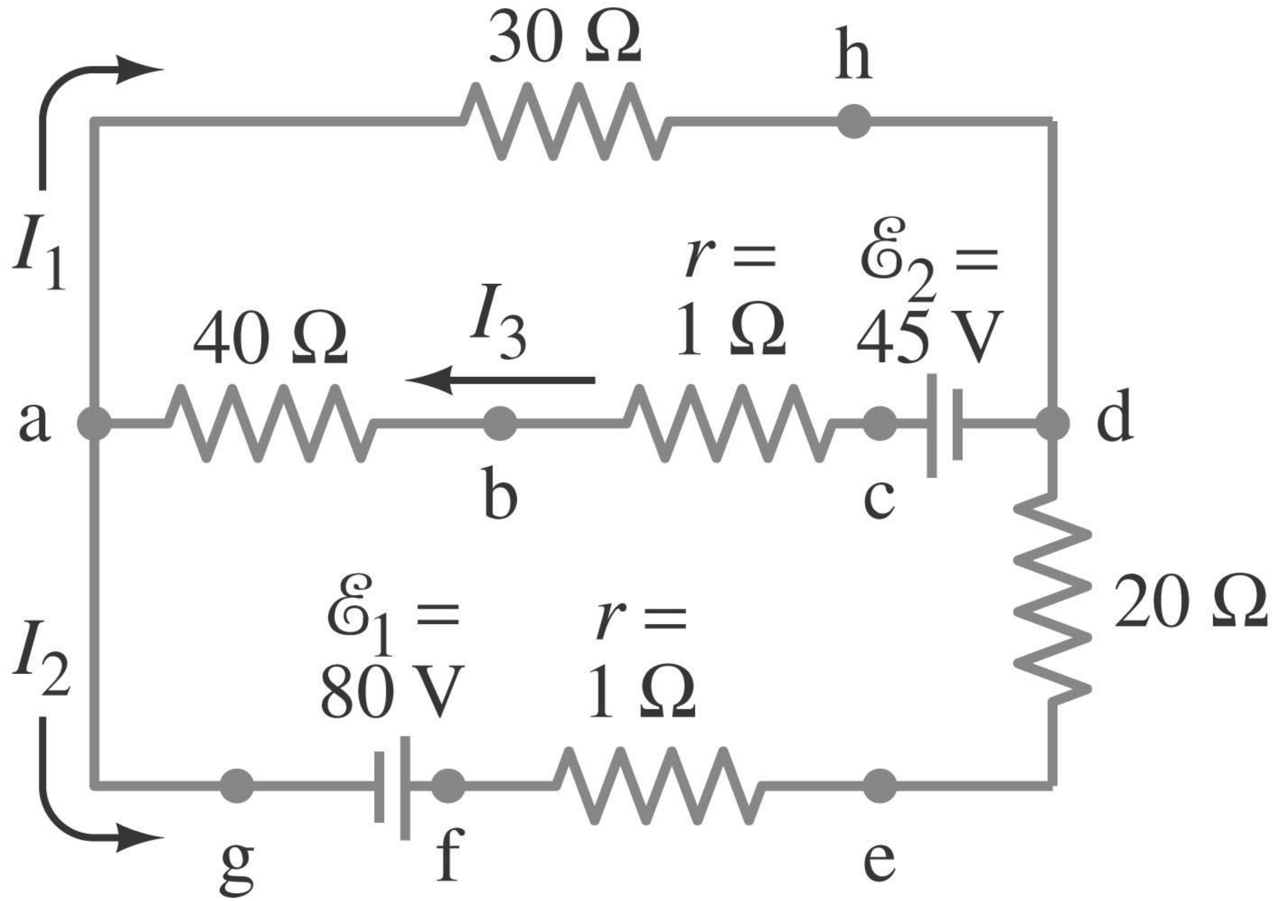
This is the current through the 6.0-Ω resistor.

26-3 Κανόνες Kirchhoff

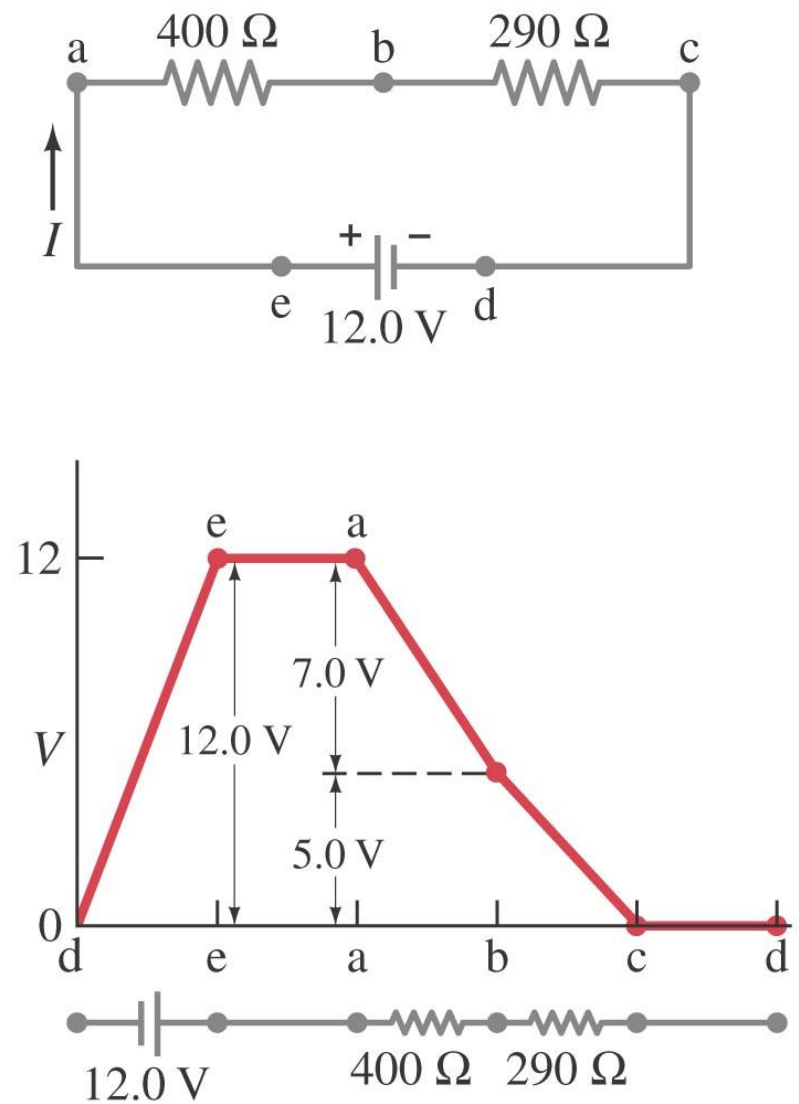
Ορισμένες φορές δεν μπορούμε να αναλύσουμε το κύκλωμα μόνο μέσα από παράλληλες και σειριακές συνδέσεις αντιστάσεων. Σε αυτές τις περιπτώσεις εφαρμόζουμε τους κανόνες του Kirchhoff's.



Κανόνας κόμβου: όσο ρεύμα μπαίνει τόσο ρεύμα φεύγει από ένα κόμβο



Κανόνας βρόχων: το άθροισμα των μεταβολών τάσεως σε ένα κλειστό βρόχο είναι μηδέν.



Εφαρμόζοντας τους κανόνες του Kirchhoff

1. Σημειώνουμε όλα τα ρεύματα και τις διευθύνσεις τους.

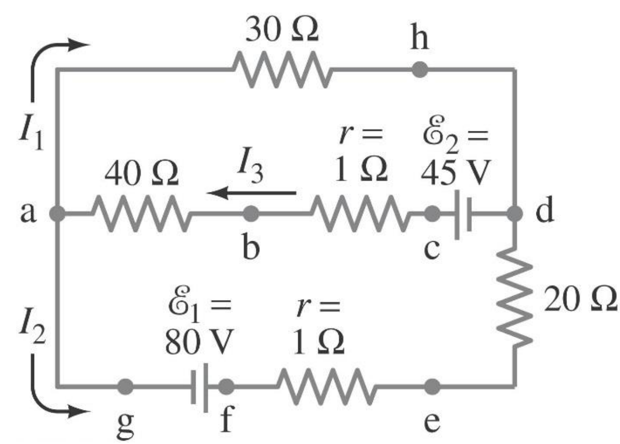
2. Αναγνωρίζουμε τους αγνώστους.

3. Εφαρμόζουμε του κανόνες Kirchhoff στους βρόχους και κόμβους.

4. Λύνουμε τις εξισώσεις που προκύπτουν ως προς τους αγνώστους. Προσοχή, χρειαζόμαστε τόσες εξισώσεις ΄σοι και οι άγνωστοι.

5. Αρνητικά ρεύματα σημαίνει αντίθετη φορά ως προς τον αρχικό μας ορισμό.

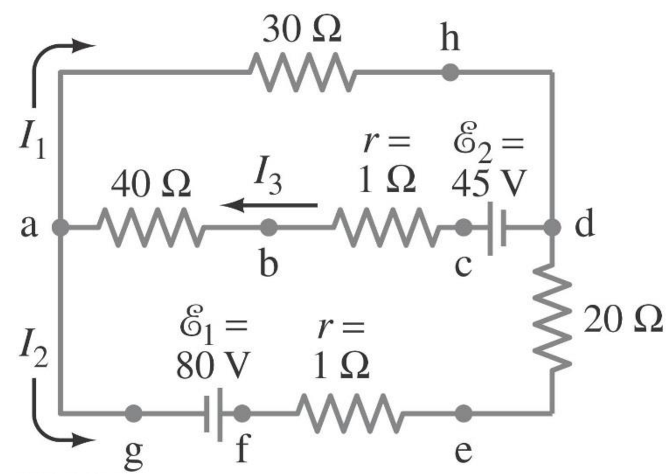
Βρείτε τα I_1, I_2, και I_3 του κυκλώματος :



2. **Identify the unknowns.** We have three unknowns and therefore we need three equations, which we get by applying Kirchhoff's junction and loop rules.
3. **Junction rule:** We apply Kirchhoff's junction rule to the currents at point a, where I_3 enters and I_2 and I_1 leave:
$$I_3 = I_1 + I_2. \qquad (a)$$
This same equation holds at point d, so we get no new information by writing an equation for point d.
4. **Loop rule:** We apply Kirchhoff's loop rule to two different closed loops. First we apply it to the upper loop ahdcba. We start (and end) at point a. From a to h we have a potential decrease $V_{ha} = -(I_1)(30\,\Omega)$. From h to d there is no change, but from d to c the potential increases by 45 V: that is, $V_{cd} = +45\text{ V}$. From c to a the potential decreases through the two resistances by an amount $V_{ac} = -(I_3)(40\,\Omega + 1\,\Omega) = -(41\,\Omega)I_3$. Thus we have $V_{ha} + V_{cd} + V_{ac} = 0$, or
$$-30I_1 + 45 - 41I_3 = 0, \qquad (b)$$
where we have omitted the units (volts and amps) so we can more easily do the algebra. For our second loop, we take the outer loop ahdefga. (We could have chosen the lower loop abcdefga instead.) Again we start at point a and have $V_{ha} = -(I_1)(30\,\Omega)$, and $V_{dh} = 0$. But when we take our positive test charge from d to e, it actually is going uphill, against the current—or at least against the *assumed* direction of the current, which is what counts in this calculation. Thus $V_{ed} = I_2(20\,\Omega)$ has a *positive* sign. Similarly, $V_{fe} = I_2(1\,\Omega)$. From f to g there is a decrease in potential of 80 V since we go from the high potential terminal of the battery to the low. Thus $V_{gf} = -80\text{ V}$. Finally, $V_{ag} = 0$, and the sum of the potential changes around this loop is
$$-30I_1 + (20 + 1)I_2 - 80 = 0. \qquad (c)$$
Our major work is done. The rest is algebra.

5. Solve the equations. We have three equations—labeled (*a*), (*b*), and (*c*)—and three unknowns. From Eq. (*c*) we have

$$I_2 = \frac{80 + 30I_1}{21} = 3.8 + 1.4I_1.$$



From Eq. (*b*) we have

$$I_3 = \frac{45 - 30I_1}{41} = 1.1 - 0.73I_1.$$

We substitute Eqs. (*d*) and (*e*) into Eq. (*a*):

$$I_1 = I_3 - I_2 = 1.1 - 0.73I_1 - 3.8 - 1.4I_1.$$

We solve for I_1, collecting terms:

$$3.1I_1 = -2.7$$
$$I_1 = -0.87 \text{ A}.$$

The negative sign indicates that the direction of I_1 is actually opposite to that initially assumed and shown in Fig. 26–13. The answer automatically comes out in amperes because all values were in volts and ohms. From Eq. (*d*) we have

$$I_2 = 3.8 + 1.4I_1 = 3.8 + 1.4(-0.87) = 2.6 \text{ A},$$

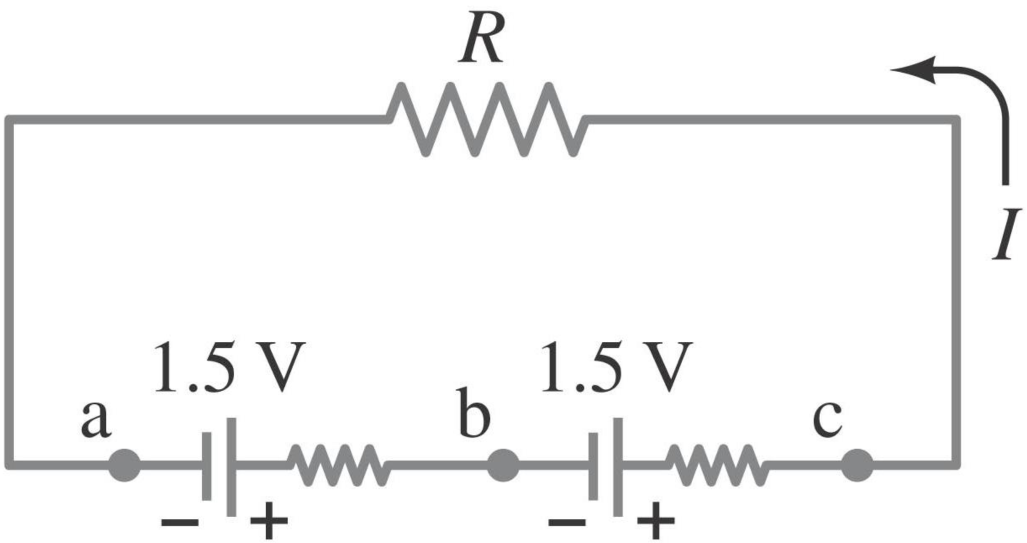
and from Eq. (*e*)

$$I_3 = 1.1 - 0.73I_1 = 1.1 - 0.73(-0.87) = 1.7 \text{ A}.$$

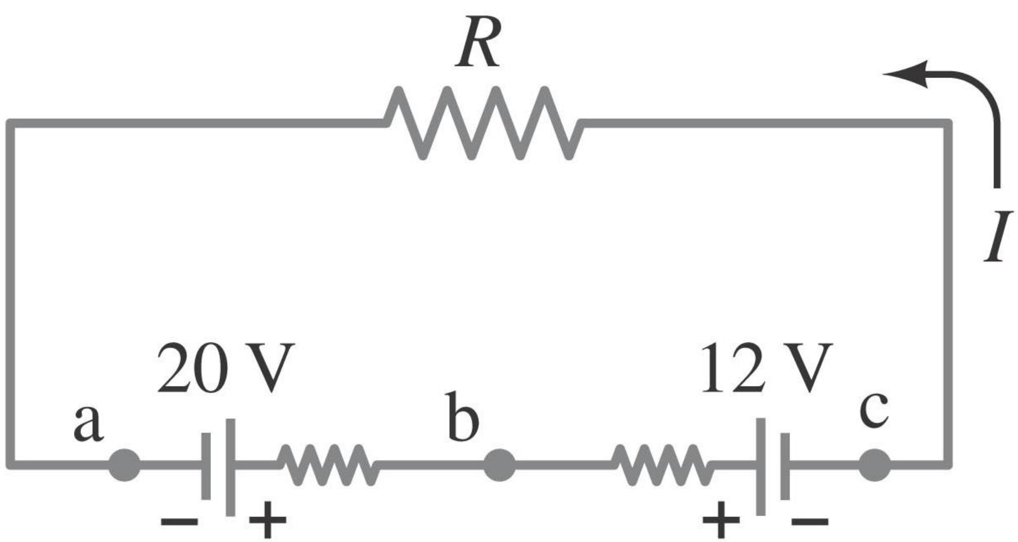
This completes the solution.

NOTE The unknowns in different situations are not necessarily currents. It might be that the currents are given and we have to solve for unknown resistance or voltage. The variables are then different, but the technique is the same.

Η τάση εξόδου για πηγές ΗΕΔ σε σειρά είναι το άθροισμα των επιμέρους ΗΕΔ.

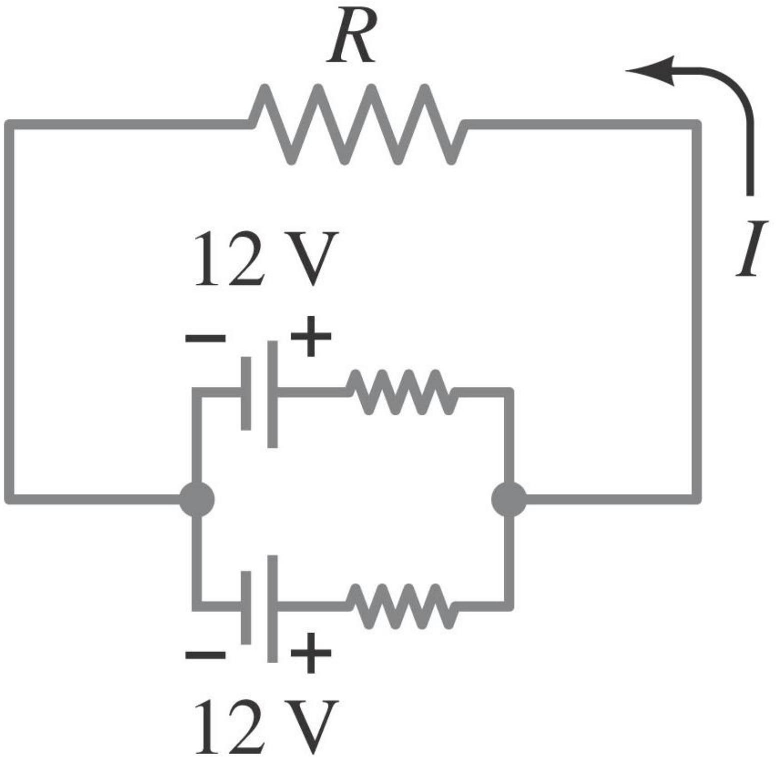


1,5V + 1,5V = 3V



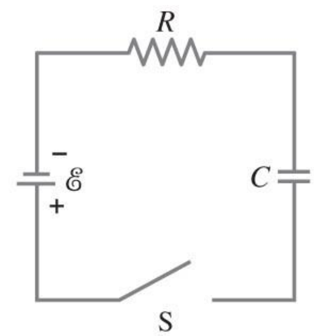
20V - 12V = 8V

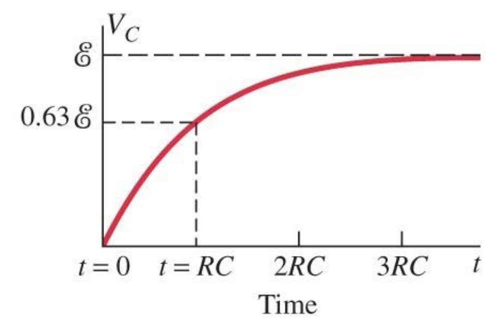
Η Παράλληλη σύνδεση μπαταριών (ΗΕΔ) έχει νόημα μόνο όταν η μπαταρίες έχουν την ίδια τάση ώστε να αυξήσουμε το ρεύμα εξόδου

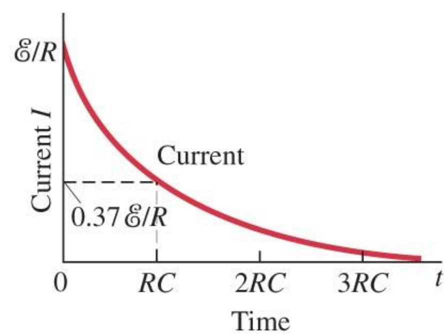


26-5 Κυκλώματα *RC* Circuits

Με το κλείσιμο του διακόπτη S, αρχίζει να «φορτώνει» ο πυκνωτής, άρα η τάση του αυξάνεται. Το ρεύμα είναι μέγιστο στην αρχή και μειώνεται σαν συνάρτηση το χρόνου.







$$\mathscr{E} = IR + \frac{Q}{C}.$$

Επειδή $I = dQ/dt$, τότε έχουμε

$$\mathscr{E} = R\frac{dQ}{dt} + \frac{1}{C}Q.$$

This equation can be solved by rearranging it:

$$\frac{dQ}{C\mathscr{E} - Q} = \frac{dt}{RC}.$$

We now integrate from $t = 0$, when $Q = 0$, to time t when a charge Q is on the capacitor:

$$\int_0^Q \frac{dQ}{C\mathscr{E} - Q} = \frac{1}{RC}\int_0^t dt$$

$$-\ln(C\mathscr{E} - Q)\Big|_0^Q = \frac{t}{RC}\Big|_0^t$$

$$-\ln(C\mathscr{E} - Q) - (-\ln C\mathscr{E}) = \frac{t}{RC}$$

$$\ln(C\mathscr{E} - Q) - \ln(C\mathscr{E}) = -\frac{t}{RC}$$

$$\ln\left(1 - \frac{Q}{C\mathscr{E}}\right) = -\frac{t}{RC}.$$

We take the exponential† of both sides

$$1 - \frac{Q}{C\mathscr{E}} = e^{-t/RC}$$

or

$$Q = C\mathscr{E}\left(1 - e^{-t/RC}\right).$$

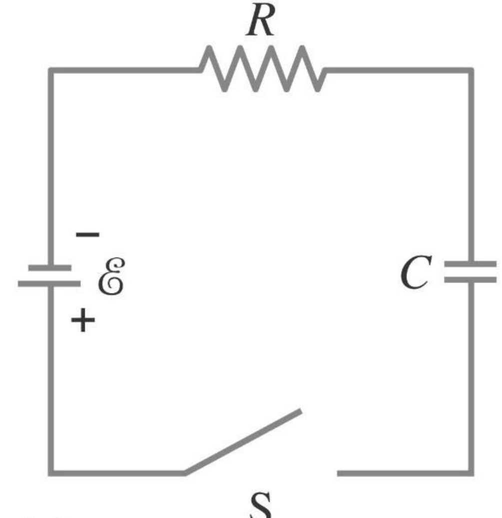
The potential difference across the capacitor is $V_C = Q/C$, so

$$V_C = \mathscr{E}\left(1 - e^{-t/RC}\right).$$

Η ποσότητα *RC* είναι η χρονική σταθερά για το κύκλωμα:

$$\tau = RC.$$

Για το κύκλωμα του διαγράμματος έχουμε C = 0.30 μF, R=20 kΩ, ΗΕΔ 12 V. Βρείτε (a) την χρονική σταθερά, (b) το μέγιστο φορτίο του πυκνωτή, (c) πόσος χρόνος απαιτείται για να φορτιστεί στο 99% της μέγιστης τιμής, (d) το ρεύμα στο ήμισυ του μέγιστου φορτίου (e) το μέγιστο ρεύμα (f) το φορτίο όταν το ρεύμα είναι 20% του μέγιστου.



SOLUTION (*a*) The time constant is $RC = (2.0 \times 10^4\,\Omega)(3.0 \times 10^{-7}\,\text{F}) = 6.0 \times 10^{-3}\,\text{s}$.
(*b*) The maximum charge would be $Q = C\mathscr{E} = (3.0 \times 10^{-7}\,\text{F})(12\,\text{V}) = 3.6\,\mu\text{C}$.
(*c*) In Eq. 26–6a, we set $Q = 0.99C\mathscr{E}$:

$$0.99C\mathscr{E} = C\mathscr{E}\left(1 - e^{-t/RC}\right),$$

or

$$e^{-t/RC} = 1 - 0.99 = 0.01.$$

Then

$$\frac{t}{RC} = -\ln(0.01) = 4.6$$

so

$$t = 4.6RC = 28 \times 10^{-3}\,\text{s}$$

or 28 ms (less than $\frac{1}{30}$ s).
(*d*) From part (*b*) the maximum charge is 3.6 μC. When the charge is half this value, 1.8 μC, the current I in the circuit can be found using the original differential equation, or Eq. 26–5:

$$I = \frac{1}{R}\left(\mathscr{E} - \frac{Q}{C}\right) = \frac{1}{2.0 \times 10^4\,\Omega}\left(12\,\text{V} - \frac{1.8 \times 10^{-6}\,\text{C}}{0.30 \times 10^{-6}\,\text{F}}\right) = 300\,\mu\text{A}.$$

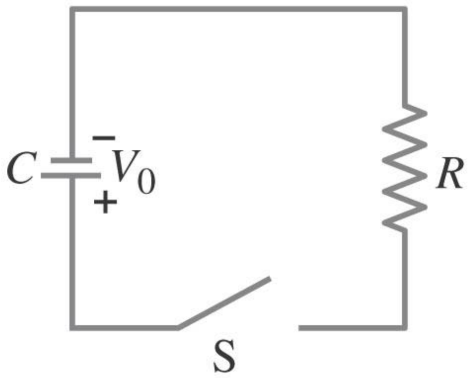
(e) The current is a maximum when there is no charge on the capacitor $(Q = 0)$:

$$I_{max} = \frac{\mathscr{E}}{R} = \frac{12\ \text{V}}{2.0 \times 10^4\ \Omega} = 600\ \mu\text{A}.$$

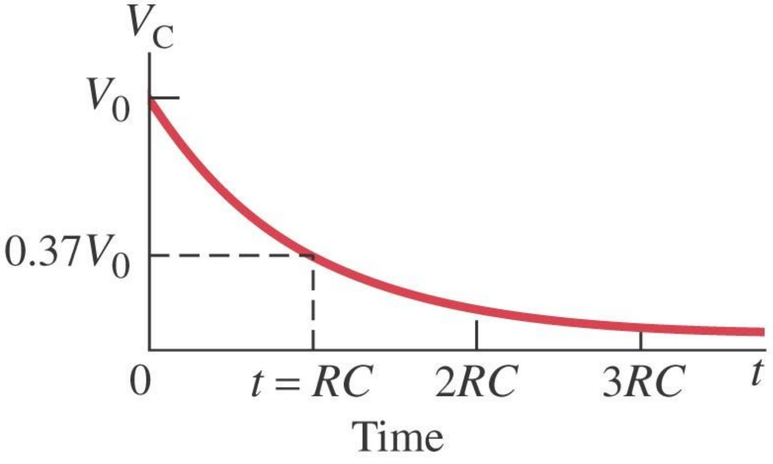
(f) Again using Eq. 26–5, now with $I = 0.20 I_{max} = 120\ \mu\text{A}$, we have

$$\begin{aligned} Q &= C(\mathscr{E} - IR) \\ &= (3.0 \times 10^{-7}\ \text{F})\left[12\ \text{V} - (1.2 \times 10^{-4}\ \text{A})(2.0 \times 10^4\ \Omega)\right] = 2.9\ \mu\text{C}. \end{aligned}$$

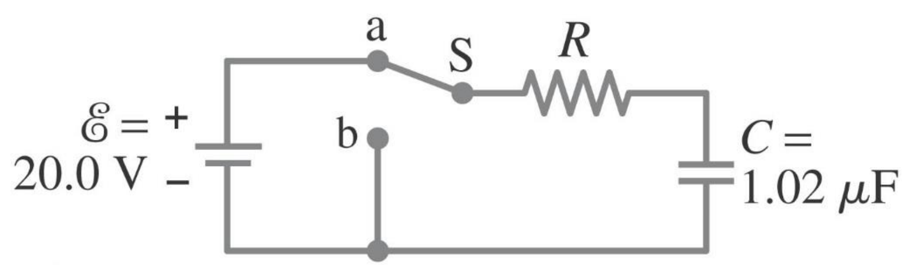
Ο χρόνος εκφόρτωσης ενός πυκνωτή μέσα από μια αντίσταση R είναι:



$$Q = Q_0 e^{-t/RC}.$$



Στο RC κύκλωμα του διαγράμματος, η μπαταρία έχει φορτώσει πλήρως των πυκνωτή, $Q_0 = C\mathcal{E}$. Στη χρονική στιγμή $t = 0$ ο διακόπτης αλλάζει θέση από το a στο b.



Η ΗΕΔ της μπαταρίας είναι 20.0 V, $C = 1.02$ μF. Το ρεύμα I φτάνει το 50% της αρχικής τιμής μέσα σε 40 μs. (a) Πόσο είναι το Q, όταν $t = 0$; (b) Βρείτε την τιμή της R (c) Βρείτε το Q όταν $t = 60$ μs?

SOLUTION (*a*) At $t = 0$,

$$Q = Q_0 = C\mathcal{E} = (1.02 \times 10^{-6}\,\mathrm{F})(20.0\,\mathrm{V}) = 2.04 \times 10^{-5}\,\mathrm{C} = 20.4\,\mu\mathrm{C}.$$

(*b*) To find R, we are given that at $t = 40\,\mu$s, $I = 0.50 I_0$. Hence

$$0.50 I_0 = I_0 e^{-t/RC}.$$

Taking natural logs on both sides $(\ln 0.50 = -0.693)$:

$$0.693 = \frac{t}{RC}$$

so

$$R = \frac{t}{(0.693)C} = \frac{(40 \times 10^{-6}\,\mathrm{s})}{(0.693)(1.02 \times 10^{-6}\,\mathrm{F})} = 57\,\Omega.$$

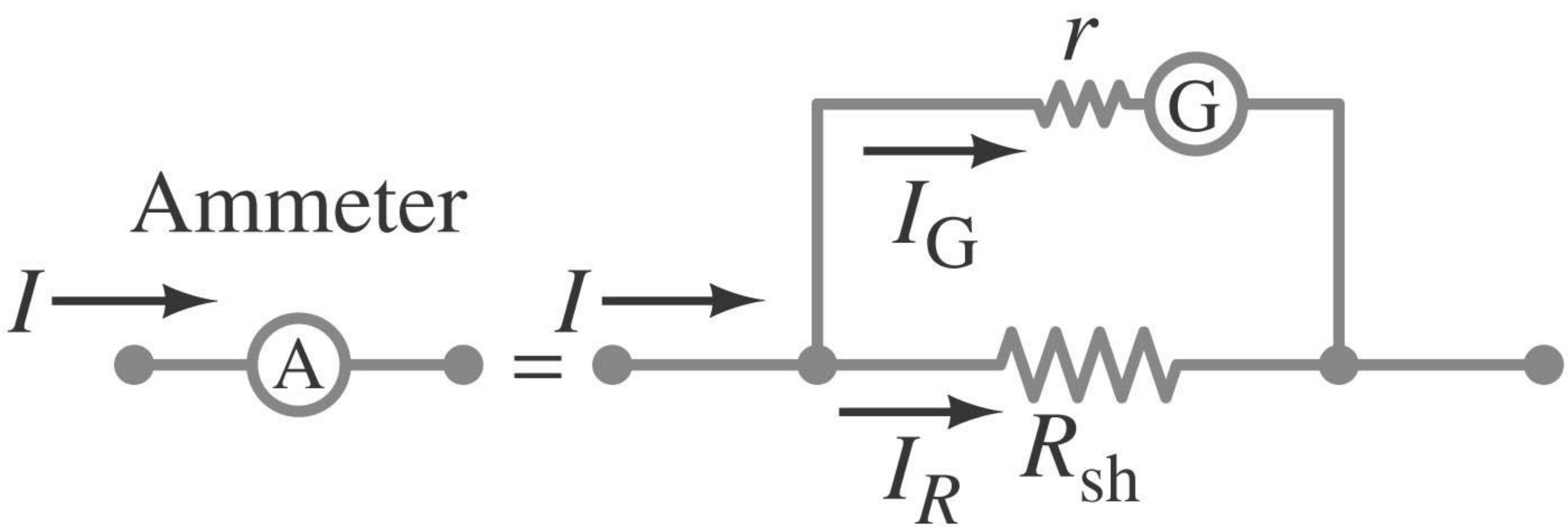
(*c*) At $t = 60\,\mu$s,

$$Q = Q_0 e^{-t/RC} = (20.4 \times 10^{-6}\,\mathrm{C}) e^{-\frac{60\times10^{-6}\,\mathrm{s}}{(57\,\Omega)(1.02\times10^{-6}\,\mathrm{F})}} = 7.3\,\mu\mathrm{C}.$$

26-7 Μέτρηση Ρεύματος και Τάσης

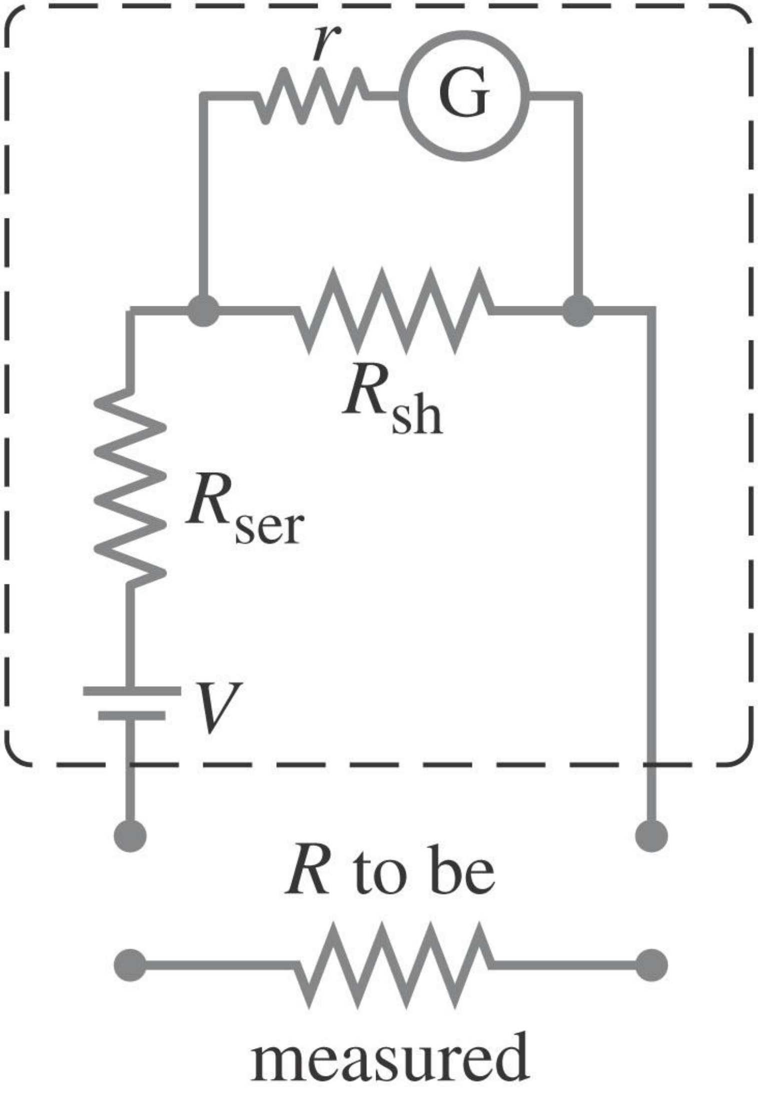
Το Αμπερόμετρο μετράει Ρεύμα: Συνδέεται σε σειρά και έτσι έχει πολύ μικρή αντίσταση

Το Βολτόμετρο μετράει Τάση: Συνδέεται παράλληλα και έχει πολύ μεγάλη αντίσταση .



26-7 Μέτρηση Αντίστασης

Η Μέτρηση αντίστασης απαιτεί μια μπαταρία για την παροχή ρεύματος.



26-7 Πολύμετρα