9. How does the energy in a capacitor change if (*a*) the potential difference is doubled, (*b*) the charge on each plate is doubled, and (*c*) the separation of the plates is doubled, as the capacitor remains connected to a battery in each case?

9. (*a*) The energy stored quadruples since the potential difference across the plates doubles and the capacitance doesn't change: $U = \frac{1}{2}CV^2$.

(*b*) The energy stored quadruples since the charge doubles and the capacitance doesn't change: $U = \frac{1}{2}\frac{Q^2}{C}$.

(*c*) If the separation between the plates doubles, the capacitance is halved. The potential difference across the plates doesn't change if the capacitor remains connected to the battery, so the energy stored is also halved: $U = \frac{1}{2}CV^2$.

5. (II) A 7.7-μF capacitor is charged by a 125-V battery (Fig. 24–20a) and then is disconnected from the battery. When this capacitor (C_1) is then connected (Fig. 24–20b) to a second (initially uncharged) capacitor, C_2, the final voltage on each capacitor is 15 V. What is the value of C_2? [*Hint*: Charge is conserved.]

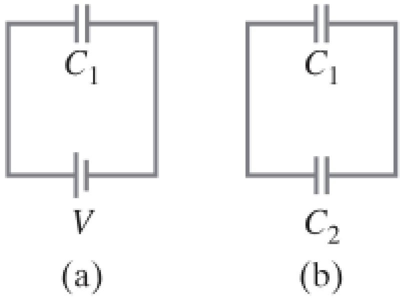


FIGURE 24–20 Problems 5 and 48.

5. After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

$$Q_{\text{Total}} = C_1 V_{1_{\text{initial}}} \qquad Q_{1_{\text{final}}} = C_1 V_{\text{final}} \qquad Q_{2_{\text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{1_{\text{final}}} + Q_{2_{\text{final}}} = (C_1 + C_2) V_{\text{final}} \rightarrow C_1 V_{1_{\text{initial}}} = (C_1 + C_2) V_{\text{final}} \rightarrow$$

$$C_2 = C_1 \left(\frac{V_{1_{\text{initial}}}}{V_{\text{final}}} - 1 \right) = (7.7 \times 10^{-6}\,\text{F}) \left(\frac{125\,\text{V}}{15\,\text{V}} - 1 \right) = 5.6 \times 10^{-5}\,\text{F} = \boxed{56\,\mu\text{F}}$$

10. (II) In a **dynamic random access memory (DRAM)** computer chip, each memory cell chiefly consists of a capacitor for charge storage. Each of these cells represents a single binary-bit value of 1 when its 35-fF capacitor $(1\text{ fF} = 10^{-15}\text{ F})$ is charged at 1.5 V, or 0 when uncharged at 0 V. (*a*) When it is fully charged, how many excess electrons are on a cell capacitor's negative plate? (*b*) After charge has been placed on a cell capacitor's plate, it slowly "leaks" off (through a variety of mechanisms) at a constant rate of 0.30 fC/s. How long does it take for the potential difference across this capacitor to decrease by 1.0% from its fully charged value? (Because of this leakage effect, the charge on a DRAM capacitor is "refreshed" many times per second.)

10. (*a*) The absolute value of the charge on each plate is given by Eq. 24-1. The plate with electrons has a net negative charge.

$$Q = CV \;\rightarrow\; N(-e) = -CV \;\rightarrow$$

$$N = \frac{CV}{e} = \frac{(35\times10^{-15}\text{F})(1.5\text{V})}{1.60\times10^{-19}\text{C}} = 3.281\times10^{5} \approx \boxed{3.3\times10^{5}\text{ electrons}}$$

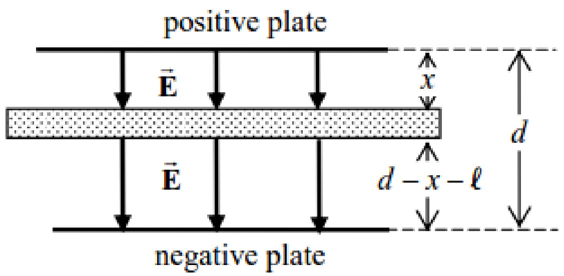
(*b*) Since the charge is directly proportional to the potential difference, a 1.0% decrease in potential difference corresponds to a 1.0% decrease in charge.

$$\Delta Q = 0.01Q \;;$$

$$\Delta t = \frac{\Delta Q}{\Delta Q/\Delta t} = \frac{0.01Q}{\Delta Q/\Delta t} = \frac{0.01CV}{\Delta Q/\Delta t} = \frac{0.01(35\times10^{-15}\text{F})(1.5\text{V})}{0.30\times10^{-15}\text{ C/s}} = 1.75\text{s} \approx \boxed{1.8\text{s}}$$

18. (II) A large metal sheet of thickness ℓ is placed between, and parallel to, the plates of the parallel-plate capacitor of Fig. 24–4. It does not touch the plates, and extends beyond their edges. (*a*) What is now the net capacitance in terms of A, d, and ℓ? (*b*) If $\ell = 0.40\,d$, by what factor does the capacitance change when the sheet is inserted?

18. (*a*) The uncharged plate will polarize so that negative charge will be drawn towards the positive capacitor plate, and positive charge will be drawn towards the negative capacitor plate. The same charge will be on each face of the plate as on the original capacitor plates. The same electric field will be in the gaps as before the plate was inserted. Use that electric field to determine the potential difference between the two original plates, and the new capacitance. Let x be the distance from one original plate to the nearest face of the sheet, and so $d-\ell-x$ is the distance from the other original plate to the other face of the sheet.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;;\; V_1 = Ex = \frac{Qx}{A\varepsilon_0} \;;\; V_2 = E(d-\ell-x) = \frac{Q(d-\ell-x)}{A\varepsilon_0}$$

$$\Delta V = V_1 + V_2 = \frac{Qx}{A\varepsilon_0} + \frac{Q(d-\ell-x)}{A\varepsilon_0} = \frac{Q(d-\ell)}{A\varepsilon_0} = \frac{Q}{C} \;\rightarrow\; C = \boxed{\varepsilon_0\frac{A}{(d-\ell)}}$$

(*b*) $$C_{\text{initial}} = \varepsilon_0\frac{A}{d} \;;\; C_{\text{final}} = \varepsilon_0\frac{A}{(d-\ell)} \;;\; \frac{C_{\text{final}}}{C_{\text{initial}}} = \frac{\varepsilon_0\dfrac{A}{(d-\ell)}}{\varepsilon_0\dfrac{A}{d}} = \frac{d}{d-\ell} = \frac{d}{d-0.40d} = \frac{1}{0.60} = \boxed{1.7}$$

29. (II) In Fig. 24–23, suppose $C_1 = C_2 = C_3 = C_4 = C$. (*a*) Determine the equivalent capacitance between points a and b. (*b*) Determine the charge on each capacitor and the potential difference across each in terms of V.

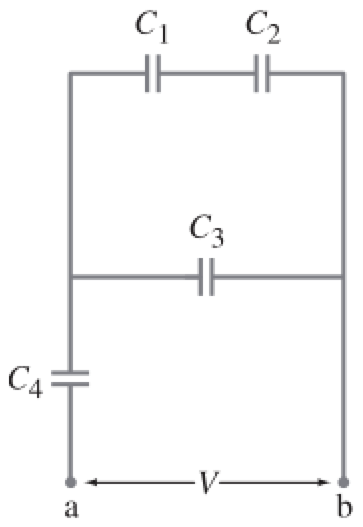


FIGURE 24–23
Problems 29 and 30.

29. (*a*) From the diagram, we see that C_1 and C_2 are in series. That combination is in parallel with C_3, and then that combination is in series with C_4. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \rightarrow C_{12} = \tfrac{1}{2}C \ ; \ C_{123} = C_{12} + C_3 = \tfrac{1}{2}C + C = \tfrac{3}{2}C \ ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \rightarrow C_{1234} = \boxed{\tfrac{3}{5}C}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V = \tfrac{3}{5}CV$. This is the charge on both of the series components of C_{1234}.

$$Q_{123} = \tfrac{3}{5}CV = C_{123}V_{123} = \tfrac{3}{2}CV_{123} \rightarrow V_{123} = \tfrac{2}{5}V$$

$$Q_4 = \tfrac{3}{5}CV = C_4V_4 \rightarrow V_4 = \tfrac{3}{5}V$$

The voltage across the equivalent capacitor C_{123} is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of C_{123} is the same as the total charge on the two components, $\tfrac{3}{5}CV$.

$$V_{123} = \tfrac{2}{5}V = V_{12} \ ; \ Q_{12} = C_{12}V_{12} = \left(\tfrac{1}{2}C\right)\left(\tfrac{2}{5}V\right) = \tfrac{1}{5}CV$$

$$V_{123} = \tfrac{2}{5}V = V_3 \ ; \ Q_3 = C_3V_3 = C\left(\tfrac{2}{5}V\right) = \tfrac{2}{5}CV$$

Finally, the charge on the equivalent capacitor C_{12} is the charge on both of the series components of C_{12}.

$$Q_{12} = \tfrac{1}{5}CV = Q_1 = C_1V_1 \rightarrow V_1 = \tfrac{1}{5}V \ ; \ Q_{12} = \tfrac{1}{5}CV = Q_2 = C_1V_2 \rightarrow V_2 = \tfrac{1}{5}V$$

Here are all the results, gathered together.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = \tfrac{1}{5}CV \ ; \ Q_3 = \tfrac{2}{5}CV \ ; \ Q_4 = \tfrac{3}{5}CV \\ V_1 = V_2 = \tfrac{1}{5}V \ ; \ V_3 = \tfrac{2}{5}V \ ; \ V_4 = \tfrac{3}{5}V \end{array}}$$

31. (II) The switch S in Fig. 24–24 is connected downward so that capacitor C_2 becomes fully charged by the battery of voltage V_0. If the switch is then connected upward, determine the charge on each capacitor after the switching.

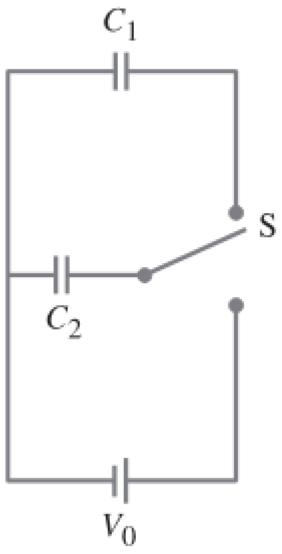
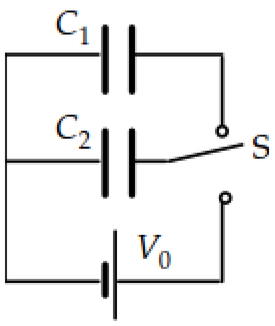


FIGURE 24–24 Problem 31.

31. When the switch is down the initial charge on C_2 is calculated from Eq. 24-1.

$$Q_2 = C_2 V_0$$

When the switch is moved up, charge will flow from C_2 to C_1 until the voltage across the two capacitors is equal.



$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \quad \rightarrow \quad Q_2' = Q_1' \frac{C_2}{C_1}$$

The sum of the charges on the two capacitors is equal to the initial charge on C_2.

$$Q_2 = Q_2' + Q_1' = Q_1' \frac{C_2}{C_1} + Q_1' = Q_1' \left(\frac{C_2 + C_1}{C_1} \right)$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$Q_1' \left(\frac{C_2 + C_1}{C_1} \right) = C_2 V_0 \quad \rightarrow \quad Q_1' = \boxed{\frac{C_1 C_2}{C_2 + C_1} V_0} \; ; \; Q_2' = Q_1' \frac{C_2}{C_1} = \boxed{\frac{C_2^2}{C_2 + C} V_0}$$

37. (II) (*a*) Determine the equivalent capacitance of the circuit shown in Fig. 24–27. (*b*) If $C_1 = C_2 = 2C_3 = 24.0\,\mu\text{F}$, how much charge is stored on each capacitor when $V = 35.0\text{ V}$?

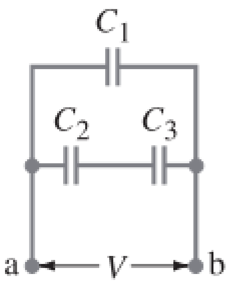


FIGURE 24–27
Problems 37, 38, and 45.

38. (II) In Fig. 24–27, let $C_1 = 2.00\,\mu\text{F}$, $C_2 = 3.00\,\mu\text{F}$, $C_3 = 4.00\,\mu\text{F}$, and $V = 24.0\text{ V}$. What is the potential difference across each capacitor?

37. (*a*) The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{eq} = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = C_1 + \left(\frac{C_3}{C_2C_3} + \frac{C_2}{C_2C_3}\right)^{-1} = C_1 + \left(\frac{C_2 + C_3}{C_2C_3}\right)^{-1} = \boxed{C_1 + \frac{C_2C_3}{C_2 + C_3}}$$

(*b*) For each capacitor, the charge is found by multiplying the capacitance times the voltage. For C_1, the full 35.0 V is across the capacitance, so $Q_1 = C_1V = (24.0\times10^{-6}\text{F})(35.0\text{V}) = \boxed{8.40\times10^{-4}\text{C}}$. The equivalent capacitance of the series combination of C_2 and C_3 has the full 35.0 V across it, and the charge on the series combination is the same as the charge on each of the individual capacitors.

$$C_{eq} = \left(\frac{1}{C} + \frac{1}{C/2}\right)^{-1} = \frac{C}{3} \qquad Q_{eq} = C_{eq}V = \tfrac{1}{3}(24.0\times10^{-6}\text{F})(35.0\text{V}) = \boxed{2.80\times10^{-4}\text{C}} = Q_2 = Q_3$$

38. From the circuit diagram, we see that C_1 is in parallel with the voltage, and so $\boxed{V_1 = 24\text{ V}}$.

Capacitors C_2 and C_3 both have the same charge, so their voltages are inversely proportional to their capacitance, and their voltages must total to 24.0 V.

$$Q_2 = Q_3 \;\rightarrow\; C_2V_2 = C_3V_3 \;;\; V_2 + V_3 = V$$

$$V_2 + \frac{C_2}{C_3}V_2 = V \;\rightarrow\; V_2 = \frac{C_3}{C_2 + C_3}V = \frac{4.00\,\mu\text{F}}{7.00\,\mu\text{F}}(24.0\text{V}) = \boxed{13.7\text{V}}$$

$$V_3 = V - V_2 = 24.0\text{V} - 13.7\text{V} = \boxed{10.3\text{V}}$$