3. (II) The momentum of a particle, in SI units, is given by $\vec{\mathbf{p}} = 4.8\,t^2\hat{\mathbf{i}} - 8.0\hat{\mathbf{j}} - 8.9\,t\hat{\mathbf{k}}$. What is the force as a function of time?

4. (II) The force on a particle of mass m is given by $\vec{\mathbf{F}} = 26\hat{\mathbf{i}} - 12\,t^2\hat{\mathbf{j}}$ where F is in N and t in s. What will be the change in the particle's momentum between $t = 1.0\,\mathrm{s}$ and $t = 2.0\,\mathrm{s}$?

8. (III) Air in a 120-km/h wind strikes head-on the face of a building 45 m wide by 65 m high and is brought to rest. If air has a mass of 1.3 kg per cubic meter, determine the average force of the wind on the building.

3. The force is the derivative of the momentum with respect to time.

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} = \frac{d\left(4.8t^2\hat{\mathbf{i}} - 8.0\vec{\mathbf{j}} - 8.9t\,\vec{\mathbf{k}}\right)}{dt} = \boxed{\left(9.6t\,\hat{\mathbf{i}} - 8.9\,\vec{\mathbf{k}}\right)\mathrm{N}}$$

4. The change in momentum is the integral of the force, since the force is the derivative of the momentum.

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} \quad\rightarrow\quad \vec{\mathbf{p}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}dt = \int_{t=1.0\mathrm{s}}^{t=2.0\mathrm{s}} \left(26\hat{\mathbf{i}} - 12t^2\vec{\mathbf{j}}\right)dt = \left(26t\hat{\mathbf{i}} - 4t^3\vec{\mathbf{j}}\right)\Big|_{t=1.0\mathrm{s}}^{t=2.0\mathrm{s}} = \boxed{\left(26\hat{\mathbf{i}} - 28\vec{\mathbf{j}}\right)\mathrm{kg\cdot m/s}}$$

8. The air is moving with an initial speed of $120\,\mathrm{km/h}\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 33.33\,\mathrm{m/s}$. Thus, in one second, a volume of air measuring 45 m x 65 m x 33.33 m will have been brought to rest. By Newton's third law, the average force on the building will be equal in magnitude to the force causing the change in momentum of the air. The mass of the stopped air is its volume times its density.

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{V\rho\Delta v}{\Delta t} = \frac{(45\,\mathrm{m})(65\,\mathrm{m})(33.33\,\mathrm{m})\left(1.3\,\mathrm{kg/m^3}\right)(33.33\,\mathrm{m/s} - 0)}{1\,\mathrm{s}} = \boxed{4.2\times10^6\,\mathrm{N}}$$

13. (II) A child in a boat throws a 5.70-kg package out horizontally with a speed of 10.0 m/s, Fig. 9–37. Calculate the velocity of the boat immediately after, assuming it was initially at rest. The mass of the child is 24.0 kg and that of the boat is 35.0 kg.

FIGURE 9–37 Problem 13.

13. The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let A represent the boat and child together, and let B represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)v = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_A = -\frac{m_B v'_B}{m_A} = -\frac{(5.70\,\text{kg})(10.0\,\text{m/s})}{(24.0\,\text{kg} + 35.0\,\text{kg})} = \boxed{-0.966\,\text{m/s}}$$

The boat and child move in the opposite direction as the thrown package, as indicated by the negative velocity.

14. (II) An atomic nucleus initially moving at 420 m/s emits an alpha particle in the direction of its velocity, and the remaining nucleus slows to 350 m/s. If the alpha particle has a mass of 4.0 u and the original nucleus has a mass of 222 u, what speed does the alpha particle have when it is emitted?

15. (II) An object at rest is suddenly broken apart into two fragments by an explosion. One fragment acquires twice the kinetic energy of the other. What is the ratio of their masses?

14. Consider the motion in one dimension, with the positive direction being the direction of motion of the original nucleus. Let A represent the alpha particle, with a mass of 4 u, and let B represent the new nucleus, with a mass of 218 u. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)v = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_A = \frac{(m_A + m_B)v - m_B v'_B}{m_A} = \frac{(222\,\text{u})(420\,\text{m/s}) - (218\,\text{u})(350\,\text{m/s})}{4.0\,\text{u}} = \boxed{4200\,\text{m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

15. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger kinetic energy.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A v'_A + m_B v'_B \rightarrow v'_B = -\frac{m_A v'_A}{m_B}$$

$$K_A = 2K_B \rightarrow \tfrac{1}{2} m_A v_A'^2 = 2\left(\tfrac{1}{2} m_B v_B'^2\right) = m_B\left(-\frac{m_A v'_A}{m_B}\right)^2 \rightarrow \frac{m_A}{m_B} = \boxed{\frac{1}{2}}$$

The fragment with the larger kinetic energy has half the mass of the other fragment.

21. (III) A 224-kg projectile, fired with a speed of 116 m/s at a 60.0° angle, breaks into three pieces of equal mass at the highest point of its arc (where its velocity is horizontal). Two of the fragments move with the same speed right after the explosion as the entire projectile had just before the explosion; one of these moves vertically downward and the other horizontally. Determine (*a*) the velocity of the third fragment immediately after the explosion and (*b*) the energy released in the explosion.

21. (*a*) For the initial projectile motion, the horizontal velocity is constant. The velocity at the highest point, immediately before the explosion, is exactly that horizontal velocity, $v_x = v_0 \cos\theta$. The explosion is an internal force, and so the momentum is conserved during the explosion. Let $\vec{\mathbf{v}}_3$ represent the velocity of the third fragment.

$$\vec{\mathbf{p}}_{\text{before}} = \vec{\mathbf{p}}_{\text{after}} \rightarrow mv_0 \cos\theta\hat{\mathbf{i}} = \tfrac{1}{3}mv_0 \cos\theta\hat{\mathbf{i}} + \tfrac{1}{3}mv_0 \cos\theta\left(-\hat{\mathbf{j}}\right) + \tfrac{1}{3}m\vec{\mathbf{v}}_3 \rightarrow$$

$$\vec{\mathbf{v}}_3 = 2v_0 \cos\theta\hat{\mathbf{i}} + v_0 \cos\theta\hat{\mathbf{j}} = 2(116\,\text{m/s})\cos 60.0°\hat{\mathbf{i}} + (116\,\text{m/s})\cos 60.0°\hat{\mathbf{j}}$$

$$= \boxed{(116\,\text{m/s})\hat{\mathbf{i}} + (58.0\,\text{m/s})\hat{\mathbf{j}}}$$

This is 130 ms at an angle of 26.6° above the horizontal.

(*b*) The energy released in the explosion is $K_{\text{after}} - K_{\text{before}}$. Note that $v_3^2 = (2v_0 \cos\theta)^2 + (v_0 \cos\theta)^2 = 5v_0^2 \cos^2\theta$.

$$K_{\text{after}} - K_{\text{before}} = \left[\tfrac{1}{2}\left(\tfrac{1}{3}m\right)(v_0 \cos\theta)^2 + \tfrac{1}{2}\left(\tfrac{1}{3}m\right)(v_0 \cos\theta)^2 + \tfrac{1}{2}\left(\tfrac{1}{3}m\right)v_3^2\right] - \tfrac{1}{2}m(v_0 \cos\theta)^2$$

$$= \tfrac{1}{2}m\left\{\left[\tfrac{1}{3}v_0^2 \cos^2\theta + \tfrac{1}{3}v_0^2 \cos^2\theta + \tfrac{1}{3}\left(5v_0^2 \cos^2\theta\right)\right] - v_0^2 \cos^2\theta\right\}$$

$$= \tfrac{1}{2}\tfrac{4}{3}mv_0^2 \cos^2\theta = \tfrac{2}{3}(224\,\text{kg})(116\,\text{m/s})^2 \cos^2 60.0° = \boxed{5.02\times10^5\,\text{J}}$$

26. (II) A 130-kg astronaut (including space suit) acquires a speed of 2.50 m/s by pushing off with his legs from a 1700-kg space capsule. (*a*) What is the change in speed of the space capsule? (*b*) If the push lasts 0.500 s, what is the average force exerted by each on the other? As the reference frame, use the position of the capsule before the push. (*c*) What is the kinetic energy of each after the push?

26. (*a*) The momentum of the astronaut–space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $v_A = v_B = 0$. We also have $v'_A = 2.50\,\text{m/s}$.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = 0 = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_B = -v'_A \frac{m_A}{m_B} = -(2.50\,\text{m/s})\frac{130\text{ kg}}{1700\text{ kg}} = -0.1912\,\text{m/s} \approx \boxed{-0.19\,\text{m/s}}$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.

(*b*) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m(v'_A - v_A)}{\Delta t} = \frac{(130\text{ kg})(2.50\,\text{m/s} - 0)}{0.500\text{ s}} = \boxed{6.5\times10^2\,\text{N}}$$

(*c*) $K_{\text{astronaut}} = \frac{1}{2}(130\,\text{kg})(2.50\,\text{m/s})^2 = \boxed{4.0\times10^2\,\text{J}}$

$K_{\text{capsule}} = \frac{1}{2}(1700\,\text{kg})(-0.1912\,\text{m/s})^2 = \boxed{31\,\text{J}}$

31. (II) (*a*) A molecule of mass m and speed v strikes a wall at right angles and rebounds back with the same speed. If the collision time is Δt, what is the average force on the wall during the collision? (*b*) If molecules, all of this type, strike the wall at intervals a time t apart (on the average) what is the average force on the wall averaged over a long time?

31. (*a*) Since the velocity changes direction, the momentum changes. Take the final velocity to be in the positive direction. Then the initial velocity is in the negative direction. The average force is the change in momentum divided by the time.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{(mv - -mv)}{\Delta t} = \boxed{2\frac{mv}{\Delta t}}$$

(*b*) Now, instead of the actual time of interaction, use the time between collisions in order to get the average force over a long time.

$$F_{\text{avg}} = \frac{\Delta p}{t} = \frac{(mv - -mv)}{t} = \boxed{2\frac{mv}{t}}$$

37. (II) A ball of mass 0.220 kg that is moving with a speed of 7.5 m/s collides head-on and elastically with another ball initially at rest. Immediately after the collision, the incoming ball bounces backward with a speed of 3.8 m/s. Calculate (*a*) the velocity of the target ball after the collision, and (*b*) the mass of the target ball.

38. (II) A ball of mass m makes a head-on elastic collision with a second ball (at rest) and rebounds with a speed equal to 0.350 its original speed. What is the mass of the second ball?

37. Let A represent the moving ball, and let B represent the ball initially at rest. The initial direction of the ball is the positive direction. We have $v_A = 7.5\,\text{m/s}$, $v_B = 0$, and $v'_A = -3.8\,\text{m/s}$.

(*a*) Use Eq. 9-8 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \rightarrow v'_B = v_A - v_B + v'_A = 7.5\,\text{m/s} - 0 - 3.8\,\text{m/s} = \boxed{3.7\,\text{m/s}}$$

(*b*) Use momentum conservation to solve for the mass of the target ball.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow$$

$$m_B = m_A \frac{(v_A - v'_A)}{(v'_B - v_B)} = (0.220\text{ kg})\frac{(7.5\,\text{m/s} - -3.8\,\text{m/s})}{3.7\,\text{m/s}} = \boxed{0.67\text{ kg}}$$

38. Use the relationships developed in Example 9-8 for this scenario.

$$v'_A = v_A\left(\frac{m_A - m_B}{m_A + m_B}\right) \rightarrow$$

$$m_B = \left(\frac{v_A - v'_A}{v'_A + v_A}\right)m_A = \left(\frac{v_A - (-0.350)v_A}{(-0.350)v_A + v_A}\right)m_A = \left(\frac{1.350}{0.650}\right)m_A = \boxed{2.08m}$$

40. (II) Show that, in general, for any head-on one-dimensional elastic collision, the speeds after collision are

$$v_B' = v_A\left(\frac{2m_A}{m_A + m_B}\right) + v_B\left(\frac{m_B - m_A}{m_A + m_B}\right)$$

and

$$v_A' = v_A\left(\frac{m_A - m_B}{m_A + m_B}\right) + v_B\left(\frac{2m_B}{m_A + m_B}\right),$$

where v_A and v_B are the initial speeds of the two objects of mass m_A and m_B.

40. Both momentum and kinetic energy are conserved in this one-dimensional collision. We start with Eq. 9-3 (for a one-dimensional setting) and Eq. 9-8.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \;;\; v_A - v_B = -(v_A' - v_B') \;\rightarrow\; v_B' = v_A - v_B + v_A'$$

Insert the last result above back into the momentum conservation equation.

$$m_A v_A + m_B v_B = m_A v_A' + m_B (v_A - v_B + v_A') = (m_A + m_B) v_A' + m_B (v_A - v_B) \;\rightarrow$$

$$m_A v_A + m_B v_B - m_B (v_A - v_B) = (m_A + m_B) v_A' \;\rightarrow\; (m_A - m_B) v_A + 2m_B v_B = (m_A + m_B) v_A' \;\rightarrow$$

$$\boxed{v_A' = v_A\left(\frac{m_A - m_B}{m_A + m_B}\right) + v_B\left(\frac{2m_B}{m_A + m_B}\right)}$$

Do a similar derivation by solving Eq. 9-8 for v_A', which gives $v_A' = v_B' - v_A + v_B$.

$$m_A v_A + m_B v_B = m_A (v_B' - v_A + v_B) + m_B v_B' = m_A(-v_A + v_B) + (m_A + m_B) v_B' \;\rightarrow$$

$$m_A v_A + m_B v_B - m_A(-v_A + v_B) = (m_A + m_B) v_B' \;\rightarrow\; 2m_A v_A + (m_B - m_A) v_B = (m_A + m_B) v_B' \;\rightarrow$$

$$\boxed{v_B' = v_A\left(\frac{2m_A}{m_A + m_B}\right) + v_B\left(\frac{m_B - m_A}{m_A + m_B}\right)}$$

45. (II) An internal explosion breaks an object, initially at rest, into two pieces, one of which has 1.5 times the mass of the other. If 7500 J is released in the explosion, how much kinetic energy does each piece acquire?

49. (II) A measure of inelasticity in a head-on collision of two objects is the *coefficient of restitution*, e, defined as

$$e = \frac{v'_A - v'_B}{v_B - v_A},$$

where $v'_A - v'_B$ is the relative velocity of the two objects after the collision and $v_B - v_A$ is their relative velocity before it. (*a*) Show that $e = 1$ for a perfectly elastic collision, and $e = 0$ for a completely inelastic collision. (*b*) A simple method for measuring the coefficient of restitution for an object colliding with a very hard surface like steel is to drop the object onto a heavy steel plate, as shown in Fig. 9–41. Determine a formula for e in terms of the original height h and the maximum height h' reached after collision.

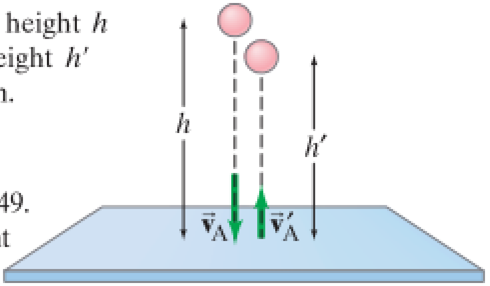


FIGURE 9–41 Problem 49. Measurement of coefficient of restitution.

43. (*a*) In Example 9-11, $K_i = \frac{1}{2}mv^2$ and $K_f = \frac{1}{2}(m+M)v'^2$. The speeds are related by

$$v' = \frac{m}{m+M}v.$$

$$\frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{\frac{1}{2}(m+M)v'^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2} = \frac{(m+M)\left(\frac{m}{m+M}v\right)^2 - mv^2}{mv^2}$$

$$= \frac{\frac{m^2v^2}{m+M} - mv^2}{mv^2} = \frac{m}{m+M} - 1 = \boxed{\frac{-M}{m+M}}$$

(*b*) For the given values, $\frac{-M}{m+M} = \frac{-380\text{ g}}{396\text{ g}} = \boxed{-0.96}$. Thus 96% of the energy is lost.

49. (*a*) For a perfectly elastic collision, Eq. 9-8 says $v_A - v_B = -(v'_A - v'_B)$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v'_A - v'_B}{v_B - v_A} = -\frac{(v_A - v_B)}{v_B - v_A} = 1.$$

For a completely inelastic collision, $v'_A = v'_B$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v'_A - v'_B}{v_B - v_A} = 0$$

(*b*) Let A represent the falling object and B represent the heavy steel plate. The speeds of the steel plate are $v_B = 0$ and $v'_B = 0$. Thus $e = -v'_A/v_A$. Consider energy conservation during the falling or rising path. The potential energy of body A at height h is transformed into kinetic energy just before it collides with the plate. Choose down to be the positive direction.

$$mgh = \frac{1}{2}mv_A^2 \quad \rightarrow \quad v_A = \sqrt{2gh}$$

The kinetic energy of body A immediately after the collision is transformed into potential energy as it rises. Also, since it is moving upwards, it has a negative velocity.

$$mgh' = \frac{1}{2}mv'^2_A \quad \rightarrow \quad v'_A = -\sqrt{2gh'}$$

Substitute the expressions for the velocities into the definition of the coefficient of restitution.

$$e = -v'_A/v_A = -\frac{-\sqrt{2gh'}}{\sqrt{2gh}} \quad \rightarrow \quad \boxed{e = \sqrt{h'/h}}$$

50. (II) A pendulum consists of a mass M hanging at the bottom end of a massless rod of length ℓ, which has a frictionless pivot at its top end. A mass m, moving as shown in Fig. 9–42 with velocity v, impacts M and becomes embedded. What is the smallest value of v sufficient to cause the pendulum (with embedded mass m) to swing clear over the top of its arc?

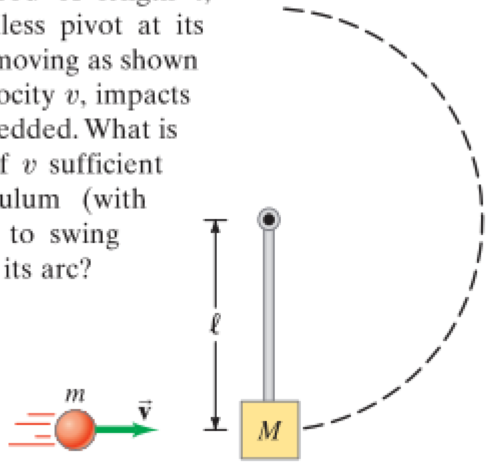


FIGURE 9–42 Problem 50.

50. The swinging motion will conserve mechanical energy. Take the zero level for gravitational potential energy to be at the bottom of the arc. For the pendulum to swing exactly to the top of the arc, the potential energy at the top of the arc must be equal to the kinetic energy at the bottom.

$$K_{\text{bottom}} = U_{\text{top}} \rightarrow \tfrac{1}{2}(m+M)V_{\text{bottom}}^2 = (m+M)g(2L) \rightarrow V_{\text{bottom}} = 2\sqrt{gL}$$

Momentum will be conserved in the totally inelastic collision at the bottom of the arc. We assume that the pendulum does not move during the collision process.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow mv = (m+M)V_{\text{bottom}} \rightarrow v = \frac{m+M}{m} = \boxed{2\frac{m+M}{m}\sqrt{gL}}$$

56. (II) Two billiard balls of equal mass move at right angles and meet at the origin of an xy coordinate system. Initially ball A is moving upward along the y axis at 2.0 m/s, and ball B is moving to the right along the x axis with speed 3.7 m/s. After the collision (assumed elastic), the second ball is moving along the positive y axis (Fig. 9–43). What is the final direction of ball A, and what are the speeds of the two balls?

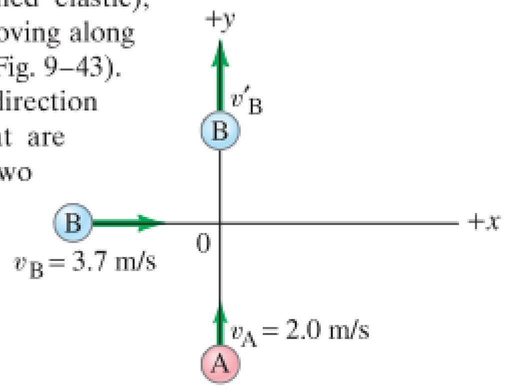


FIGURE 9–43 Problem 56. (Ball A after the collision is not shown.)

56. Write momentum conservation in the x and y directions, and kinetic energy conservation. Note that both masses are the same. We allow $\vec{\mathbf{v}}'_A$ to have both x and y components.

$$p_x: \quad mv_B = mv'_{Ax} \rightarrow v_B = v'_{Ax}$$

$$p_y: \quad mv_A = mv'_{Ay} + mv'_B \rightarrow v_A = v'_{Ay} + v'_B$$

$$K: \quad \tfrac{1}{2}mv_A^2 + \tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_A'^2 + \tfrac{1}{2}mv_B'^2 \rightarrow v_A^2 + v_B^2 = v_A'^2 + v_B'^2$$

Substitute the results from the momentum equations into the kinetic energy equation.

$$(v'_{Ay} + v'_B)^2 + (v'_{Ax})^2 = v_A'^2 + v_B'^2 \rightarrow v_{Ay}'^2 + 2v_{Ay}'^2 v'_B + v_B'^2 + v_{Ay}'^2 = v_A'^2 + v_B'^2 \rightarrow$$

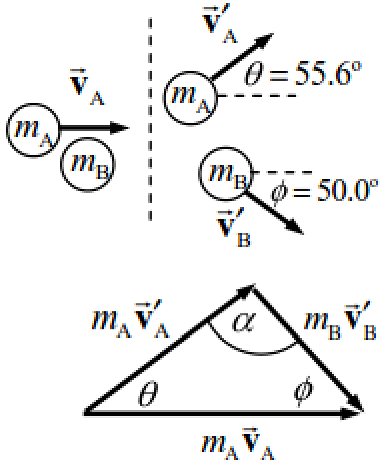
$$v_A'^2 + 2v_{Ay}'^2 v'_B + v_B'^2 = v_A'^2 + v_B'^2 \rightarrow 2v_{Ay}'^2 v'_B = 0 \rightarrow v'_{Ay} = 0 \text{ or } v'_B = 0$$

Since we are given that $v'_B \neq 0$, we must have $v'_{Ay} = 0$. This means that the final direction of A is the $\boxed{x \text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$$v'_A = v'_{Ax} = v_B = \boxed{3.7\,\text{m/s}} \qquad v'_B = v_A = \boxed{2.0\,\text{m/s}}$$

59. (III) A neon atom ($m = 20.0\,\text{u}$) makes a perfectly elastic collision with another atom at rest. After the impact, the neon atom travels away at a 55.6° angle from its original direction and the unknown atom travels away at a −50.0° angle. What is the mass (in u) of the unknown atom? [*Hint*: You could use the law of sines.]

59. Let A represent the incoming neon atom, and B represent the target atom. A momentum diagram of the collision looks like the first figure. The figure can be re-drawn as a triangle, the second figure, since $m_A \vec{\mathbf{v}}_A = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$. Write the law of sines for this triangle, relating each final momentum magnitude to the initial momentum magnitude.



$$\frac{m_A v'_A}{m_A v_A} = \frac{\sin\phi}{\sin\alpha} \rightarrow v'_A = v_A \frac{\sin\phi}{\sin\alpha}$$

$$\frac{m_B v'_B}{m_A v_A} = \frac{\sin\theta}{\sin\alpha} \rightarrow v'_B = v_A \frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}$$

The collision is elastic, so write the kinetic energy conservation equation, and substitute the results from above. Also note that $\alpha = 180.0 - 55.6^\circ - 50.0^\circ = 74.4^\circ$.

$$\tfrac{1}{2} m_A v_A^2 = \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2 \rightarrow m_A v_A^2 = m_A\left(v_A \frac{\sin\phi}{\sin\alpha}\right)^2 + m_B\left(v_A \frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}\right)^2 \rightarrow$$

$$m_B = \frac{m_A \sin^2\theta}{\sin^2\alpha - \sin^2\phi} = \frac{(20.0\,\text{u})\sin^2 55.6^\circ}{\sin^2 74.4 - \sin^2 50.0^\circ} = \boxed{39.9\,\text{u}}$$

63. (I) The distance between a carbon atom ($m = 12\,\text{u}$) and an oxygen atom ($m = 16\,\text{u}$) in the CO molecule is 1.13×10^{-10} m. How far from the carbon atom is the center of mass of the molecule?

63. Choose the carbon atom as the origin of coordinates.

$$x_{\text{CM}} = \frac{m_C x_C + m_O x_O}{m_C + m_O} = \frac{(12\text{ u})(0) + (16\text{ u})(1.13\times10^{-10}\text{ m})}{12\text{ u} + 16\text{ u}} = \boxed{6.5\times10^{-11}\text{ m}} \text{ from the C atom.}$$

72. (II) Mass $M_A = 35$ kg and mass $M_B = 25$ kg. They have velocities (in m/s) $\vec{\mathbf{v}}_A = 12\hat{\mathbf{i}} - 16\hat{\mathbf{j}}$ and $\vec{\mathbf{v}}_B = -20\hat{\mathbf{i}} + 14\hat{\mathbf{j}}$. Determine the velocity of the center of mass of the system.

72. From Eq. 9-15, we see that $\vec{\mathbf{v}}_{\text{CM}} = \frac{1}{M}\sum m_i \vec{\mathbf{v}}_i$.

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{(35\,\text{kg})(12\hat{\mathbf{i}} - 16\hat{\mathbf{j}})\,\text{m/s} + (25\,\text{kg})(-20\hat{\mathbf{i}} + 14\hat{\mathbf{j}})\,\text{m/s}}{(35\,\text{kg} + 25\,\text{kg})}$$

$$= \frac{[(35)(12) - (25)(20)]\hat{\mathbf{i}}\,\text{kg}\cdot\text{m/s} + [(35)(-12) + (25)(24)]\hat{\mathbf{j}}\,\text{kg}\cdot\text{m/s}}{(60\,\text{kg})}$$

$$= \frac{-80\hat{\mathbf{i}}\,\text{kg}\cdot\text{m/s} - 210\hat{\mathbf{j}}\,\text{kg}\cdot\text{m/s}}{(60\,\text{kg})} = \boxed{-1.3\hat{\mathbf{i}}\,\text{m/s} - 3.5\hat{\mathbf{j}}\,\text{m/s}}$$

85. A novice pool player is faced with the corner pocket shot shown in Fig. 9–48. Relative dimensions are also shown. Should the player worry that this might be a "scratch shot," in which the cue ball will also fall into a pocket? Give details. Assume equal mass balls and an elastic collision.

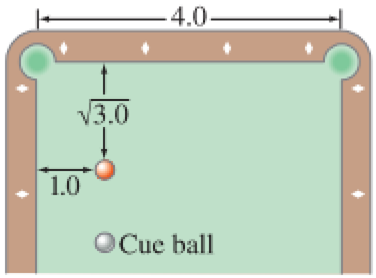
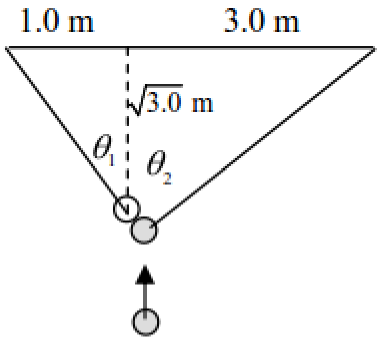


FIGURE 9–48
Problem 85.

85. It is proven in the solution to problem 61 that in an elastic collision between two objects of equal mass, with the target object initially stationary, the angle between the final velocities of the objects is 90°. For this specific circumstance, see the diagram. We assume that the target ball is hit "correctly" so that it goes in the pocket. Find θ_1 from the geometry of the "left' triangle: $\theta_1 = \tan^{-1}\frac{1.0}{\sqrt{3.0}} = 30°$. Find θ_2 from the geometry of the "right" triangle: $\theta_2 = \tan^{-1}\frac{3.0}{\sqrt{3.0}} = 60°$. Since the balls will separate at a 90° angle, if the target ball goes in the pocket, this does appear to be a $\boxed{\text{good possibility of a scratch shot}}$.



99. Two balls, of masses $m_A = 45\,\text{g}$ and $m_B = 65\,\text{g}$, are suspended as shown in Fig. 9–52. The lighter ball is pulled away to a 66° angle with the vertical and released. (*a*) What is the velocity of the lighter ball before impact? (*b*) What is the velocity of each ball after the elastic collision? (*c*) What will be the maximum height of each ball after the elastic collision?

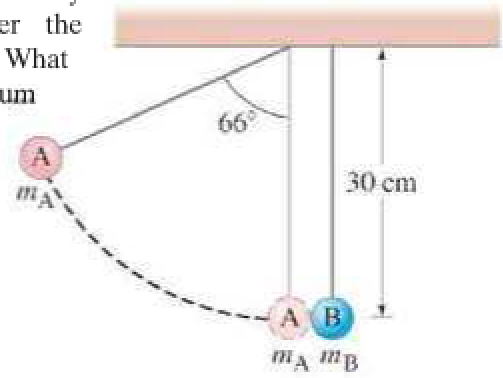
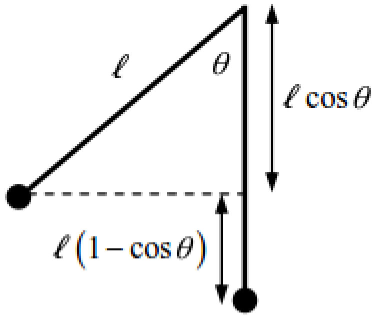


FIGURE 9–52 Problem 99.

99. (*a*) Conservation of mechanical energy can be used to find the velocity of the lighter ball before impact. The potential energy of the ball at the highest point is equal to the kinetic energy of the ball just before impact. Take the lowest point in the swing as the zero location for gravitational potential energy.



$$E_{\text{initial}} = E_{\text{final}} \rightarrow m_A g\ell(1-\cos\theta) = \tfrac{1}{2}m_A v_A^2 \rightarrow$$

$$v_A = \sqrt{2g\ell(1-\cos\theta)} = \sqrt{2(9.80\,\text{m/s}^2)(0.30\,\text{m})(1-\cos 66^\circ)}$$

$$= 1.868\,\text{m/s} \approx \boxed{1.9\,\text{m/s}}$$

(*b*) This is an elastic collision with a stationary target. Accordingly, the relationships developed in Example 9-8 are applicable.

$$v_A' = v_A\left(\frac{m_A - m_B}{m_A + m_B}\right) = (1.868\,\text{m/s})\left(\frac{0.045\,\text{kg} - 0.065\,\text{kg}}{0.045\,\text{kg} + 0.065\,\text{kg}}\right) = -0.3396\,\text{m/s} = \boxed{-0.34\,\text{m/s}}$$

$$v_B' = v_A\left(\frac{2m_A}{m_A + m_B}\right) = (1.868\,\text{m/s})\left(\frac{2(0.045\,\text{kg})}{0.045\,\text{kg} + 0.065\,\text{kg}}\right) = 1.528\,\text{m/s} = \boxed{1.5\,\text{m/s}}$$

(*c*) We can again use conservation of energy for each ball after the collision. The kinetic energy of each ball immediately after the collision will become gravitational potential energy as each ball rises.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow \tfrac{1}{2}mv^2 = mgh \rightarrow h = \frac{v^2}{2g}$$

$$h_A = \frac{v_A^2}{2g} = \frac{(-0.3396\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = \boxed{5.9\times10^{-3}\,\text{m}} \;;\; h_B = \frac{v_B^2}{2g} = \frac{(1.528\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = \boxed{0.12\,\text{m}}$$

107. In a physics lab, a cube slides down a frictionless incline as shown in Fig. 9–57 and elastically strikes another cube at the bottom that is only one-half its mass. If the incline is 35 cm high and the table is 95 cm off the floor, where does each cube land? [*Hint*: Both leave the incline moving horizontally.]

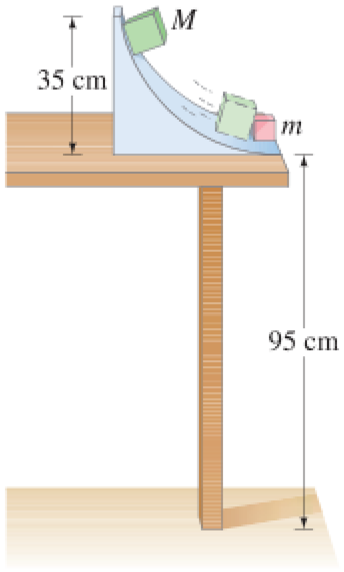


FIGURE 9–57
Problem 107.

107. Let A represent the cube of mass M and B represent the cube of mass m. Find the speed of A immediately before the collision, v_A, by using energy conservation.

$$Mgh = \tfrac{1}{2}Mv_A^2 \rightarrow v_A = \sqrt{2gh} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.35\text{ m}\right)} = 2.619\,\text{m/s}$$

Use Eq. 9-8 for elastic collisions to obtain a relationship between the velocities in the collision. We have $v_B = 0$ and $M = 2m$.

$$v_A - v_B = -\left(v_A' - v_B'\right) \rightarrow v_B' = v_A + v_A'$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \rightarrow m_A v_A = m_A v_A' + m_B\left(v_A + v_A'\right) \rightarrow$$

$$2mv_A = 2mv_A' + m\left(v_A + v_A'\right) \rightarrow v_A' = \frac{v_A}{3} = \frac{\sqrt{2gh}}{3} = \frac{\sqrt{2\left(9.80\,\text{m/s}^2\right)\left(0.35\,\text{m}\right)}}{3} = 0.873\,\text{m/s}$$

$$v_B' = v_A + v_A' = \tfrac{4}{3}v_A = 3.492\,\text{m/s}$$

Each mass is moving horizontally initially after the collision, and so each has a vertical velocity of 0 as they start to fall. Use constant acceleration Eq. 2-12b with down as positive and the table top as the vertical origin to find the time of fall.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow H = 0 + 0 + \tfrac{1}{2}gt^2 \rightarrow t = \sqrt{2H/g}$$

Each cube then travels a horizontal distance found by $\Delta x = v_x \Delta t$.

$$\Delta x_m = v_A'\Delta t = \frac{\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{2}{3}\sqrt{hH} = \tfrac{2}{3}\sqrt{\left(0.35\,\text{m}\right)\left(0.95\,\text{m}\right)} = 0.3844\text{ m} \approx \boxed{0.38\,\text{m}}$$

$$\Delta x_M = v_B'\Delta t = \frac{4\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{8}{3}\sqrt{hH} = \tfrac{8}{3}\sqrt{\left(0.35\,\text{m}\right)\left(0.95\,\text{m}\right)} = 1.538\text{ m} \approx \boxed{1.5\,\text{m}}$$

1. Το μόριο του μεθυλοβρωμίου (CH_3Br) διεγείρεται με ακτινοβολία λέιζερ ενέργειας 5,45 eV. Το μόριο σπάει και παράγει μεθυλική ρίζα (CH3) και ατομικό Βρώμιο (^{79}Br). Η εσωτερική ενέργεια του CH_3 είναι 0,62 eV, και η σπουδή του Br 8,51x10^2 m/s

(α) Βρείτε την ενέργεια του δεσμού D_0 του CH_3Br.

(β) Εάν η διεύθυνση της ταχύτητας του Br είναι $15\hat{i}+6\hat{j}-\hat{k}$ βρείτε την διεύθυνση της ταχύτητας της μεθυλικής ρίζας (δώστε το αντίστοιχο διάνυσμα).

(γ) Εάν επαναλαμβάναμε το πείραμα με το ισότοπο 81 του Βρωμίου, πόση θα είναι η σπουδή του ^{81}Br εάν η μόνη διαφορά είναι η μάζα.

Δίδονται 1eV=1,6x10^{-19} J, 1amu=1,66x10^{-27}kg, m(C)=12 amu, m(H)=1 amu, m(^{79}Br)=79 amu, m(^{81}Br)=81 amu

1. Από την Κινητική ενέργεια του Br βρίσκουμε την E_a την ενέργειας διαθέσιμη στα «φωτο-θραύσματα»

$$KE(Br)=\frac{m(CH_3)}{m(CH_3Br)}E_a=\frac{1}{2}m(Br)v^2(Br)$$

$$E_a=\frac{m(CH_3Br)m(Br)v^2(Br)}{2m(CH_3)}=\frac{94amu\times 79amu\times 1{,}66\frac{kg}{amu}\times 10^{-27}(8{,}51\times 10^2 m/s)^2}{2\times 15amu}$$

$$=2{,}97\times 10^{-19}J=\frac{2{,}97\times 10^{-19}J}{1{,}6\times 10^{-19}J/eV}$$

$$=1{,}86eV$$

Εάν $E_{INT}(CH_3)$ είναι η εσωτερική ενέργειας του CH3 και E(laser) η ενέργεια του λέιζερ τότε έχουμε

$$Ea=E(laser)-D_0-E_{INT}(CH3)\Rightarrow$$

$$D_0=E(laser)-Ea-E_{INT}(CH3)$$

$$=5{,}45-1{,}86-0{,}62$$

$$=2{,}97eV$$

$$=4{,}75\times 10^{-19}J$$

(β) $-15\hat{i}-6\hat{j}+\hat{k}$ αφού πρέπει να έχουμε διατήρηση της ορμής το CH_3 θα κινηθεί αντίθετα από το Br.

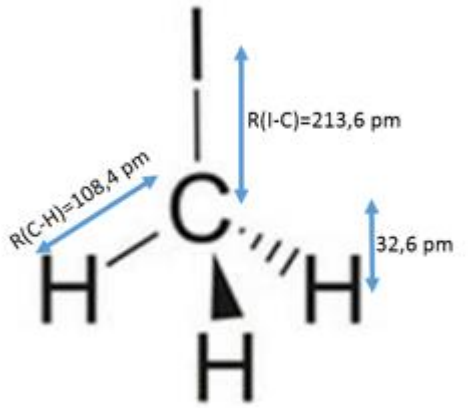
(γ) Εφόσον το μόνο η μάζα αλλάζει είναι η μάζα έχουμε

$$\left.\begin{aligned}\frac{m(CH_3)}{M(CH_3{}^{79}Br)}E_a=\frac{1}{2}m(^{79}Br)v^2(^{79}Br)\\ \frac{m(CH_3)}{M(CH_3{}^{81}Br)}E_a=\frac{1}{2}m(^{81}Br)v^2(^{81}Br)\end{aligned}\right\}$$

$$\frac{M(CH_3{}^{81}Br)}{M(CH_3{}^{79}Br)}=\frac{m(^{79}Br)v^2(^{79}Br)}{m(^{81}Br)v^2(^{81}Br)}\Rightarrow v(^{81}Br)=\sqrt{\frac{m(^{79}Br)M(CH_3{}^{79}Br)}{m(^{81}Br)M(CH_3{}^{81}Br)}}v(^{79}Br)=\sqrt{\frac{79\times 94}{81\times 95}}\times 8{,}51\times 10^2 m/s$$

$$v(^{81}Br)=8{,}36\times 10^2 m/s$$

Βρείτε το κέντρο της μάζας. Προτεινόμενη μέθοδος: Βρείτε πρώτα το κέντρο της μάζας για τα 3 υδρογόνα KM_H. Στη συνέχεια βρείτε το κέντρο της μάζας για του ζεύγους C-KM_H, ας το ονομάσουμε KM_{CH}, και συνέχεια το κέντρο μάζας μεταξύ I και KM_{CH}, όπου είναι και το κέντρο μάζας του μορίου. Στο σχήμα, η απόσταση 32,6 pm είναι η απόσταση του C από το επίπεδο που ορίζουν τα 3 H. (2 pts)

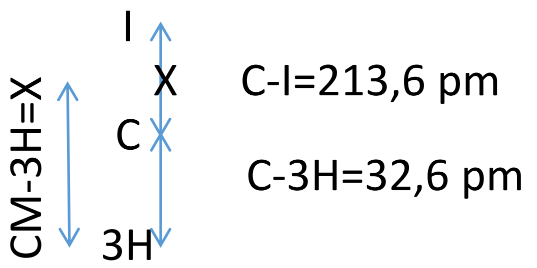


Το μόριο ακτινοβολείται με φως ενέργειας 5 eV και σπάει σε I και CH_3. Εάν η ενέργεια του δεσμού είναι 2,42 eV και η εσωτερική ενέργεια του προϊόντος CH_3 0,75 eV, βρείτε την σπουδή (μέτρο της ταχύτητας) του I και του CH_3. Δείξτε σε σχήμα ή περιγράψτε τις διευθύνσεις και την φορά των ταχυτήτων. (4 pts)

Δίδονται 1eV=1,6x10-19 J=96,5 kJ/mol, 1 amu=1,66x10-27kg, m(H)=1 amu, m(C)=12 amu. m(I)=127 amu.

Κέντρο της μάζας: Τα H βρίσκονται στο ίδιο επίπεδο, και δημιουργούν ισόπλευρο τρίγωνο. Το κέντρο της μάζας τους βρίσκεται στο κέντρο του τριγώνου, δηλ. πάνω στο διεύθυνση που δημιουργούν τα άτομα C-I. Επομένως το μόριο μπορεί να αντιπροσωπευτεί με τον γραμμικό μόριο

Εάν θεωρήσουμε την αρχή των αξόνων πάνω στο 3H τότε το κέντρο της μάζας βρίσκεται στο σημείο X τότε έχουμε



$$X(m_c + 3m_H + m_I) = (32{,}6pm)m_c + (0)3\,m_H + (213{,}6pm + 32{,}6pm)m_I$$

$$X = \frac{32{,}6 \cdot 12 + 246.2 \cdot 127}{12 + 3 + 127} pm = 222{,}9\ pm$$

Η διαθέσιμη ενέργεια στα ατομικά/μοριακά θραύσματα είναι

$$\mathrm{E}_{\alpha} = \mathrm{E}_{laser} - \mathrm{E}_{\delta\varepsilon\sigma\mu o\acute{\upsilon}} - \mathrm{E}_{\varepsilon\sigma\omega\tau\varepsilon\rho\iota\kappa} = 5 - 2.42 - 0{,}75\ eV = 1.83\ eV$$

$$KE(I) = \frac{m_{CH3}}{m_{CH3I}} E_a = \frac{15}{142} 1{,}83eV = 0{,}193eV$$

$$KE(CH_3) = \frac{m_I}{m_{CH3I}} E_a = \frac{127}{142} 1{,}83eV = 1{,}64eV$$

$$v(I) = \frac{1}{1{,}02}\sqrt{\frac{2KE}{m}} = \frac{1}{1{,}02}\sqrt{\frac{2 \cdot 0{,}193}{127}} \frac{cm}{\mu s} = 0{,}0540 \frac{cm}{\mu s} = 0{,}0540 \frac{10^{-2} m}{10^{-6} s} = 5{,}40\ 10^2 m/s$$

$$v(CH_3) = \frac{1}{1{,}02}\sqrt{\frac{2KE}{m}} = \frac{1}{1{,}02}\sqrt{\frac{2 \cdot 1{,}64}{15}} \frac{cm}{\mu s} = 0{,}458 \frac{cm}{\mu s} = 0{,}458 \frac{10^{-2} m}{10^{-6} s} = 45{,}8\ 10^2 m/s$$