*28. Name the type of equilibrium for each position of the balls in Fig. 8–30.

28. A is a point of unstable equilibrium, B is a point of stable equilibrium, and C is a point of neutral equilibrium.

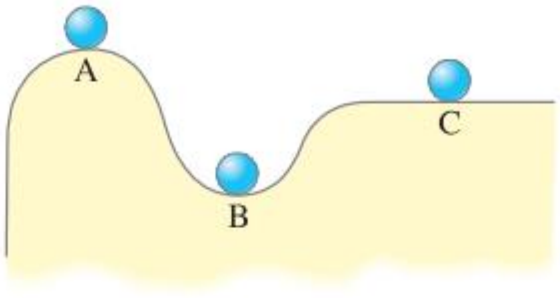


FIGURE 8–30 Question 28.

3. (II) A spring with $k = 63\ \text{N/m}$ hangs vertically next to a ruler. The end of the spring is next to the 15-cm mark on the ruler. If a 2.5-kg mass is now attached to the end of the spring, where will the end of the spring line up with the ruler marks?

3. The spring will stretch enough to hold up the mass. The force exerted by the spring will be equal to the weight of the mass.

$$mg = k(\Delta x) \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(2.5\ \text{kg})(9.80\ \text{m/s}^2)}{63\ \text{N/m}} = 0.39\ \text{m}$$

Thus the ruler reading will be $\boxed{39\ \text{cm} + 15\ \text{cm} = 54\ \text{cm}}$.

7. (II) A particular spring obeys the force law $\vec{\mathbf{F}} = (-kx + ax^3 + bx^4)\hat{\mathbf{i}}$. (*a*) Is this force conservative? Explain why or why not. (*b*) If it is conservative, determine the form of the potential energy function.

12. (I) Jane, looking for Tarzan, is running at top speed (5.0 m/s) and grabs a vine hanging vertically from a tall tree in the jungle. How high can she swing upward? Does the length of the vine affect your answer?

7. (*a*) This force is conservative, because the work done by the force on an object moving from an initial position (x_1) to a final position (x_2) depends only on the endpoints.

$$W = \int_{x_1}^{x_2} \vec{\mathbf{F}}\cdot d\vec{\mathbf{l}} = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} \left(-kx + ax^3 + bx^4\right) dx = \left(-\tfrac{1}{2}kx^2 + \tfrac{1}{4}ax^4 + \tfrac{1}{5}bx^5\right)_{x_1}^{x_2}$$

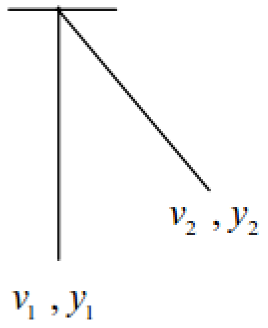
$$= \left(-\tfrac{1}{2}kx_2^2 + \tfrac{1}{4}ax_2^4 + \tfrac{1}{5}bx_2^5\right) - \left(-\tfrac{1}{2}kx_1^2 + \tfrac{1}{4}ax_1^4 + \tfrac{1}{5}bx_1^5\right)$$

The expression for the work only depends on the endpoints.

(*b*) Since the force is conservative, there is a potential energy function U such that $F_x = -\dfrac{\partial U}{\partial x}$.

$$F_x = \left(-kx + ax^3 + bx^4\right) = -\frac{\partial U}{\partial x} \rightarrow \boxed{U(x) = \tfrac{1}{2}kx^2 - \tfrac{1}{4}ax^4 - \tfrac{1}{5}bx^5 + C}$$

12. The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, and so can do no work – the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for gravitational potential energy $(y = 0)$. We have $v_1 = 5.0\,\text{m/s}$, $y_1 = 0$, and $v_2 = 0$ (top of swing). Solve for y_2, the height of her swing.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \rightarrow$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(5.0\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = 1.276\,\text{m} \approx \boxed{1.3\,\text{m}}$$

$\boxed{\text{No}}$, the length of the vine does not enter into the calculation, unless the vine is less than 0.65 m long. If that were the case, she could not rise 1.3 m high.

16. (II) A 72-kg trampoline artist jumps vertically upward from the top of a platform with a speed of 4.5 m/s. (*a*) How fast is he going as he lands on the trampoline, 2.0 m below (Fig. 8–31)? (*b*) If the trampoline behaves like a spring of spring constant 5.8×10^4 N/m, how far does he depress it?

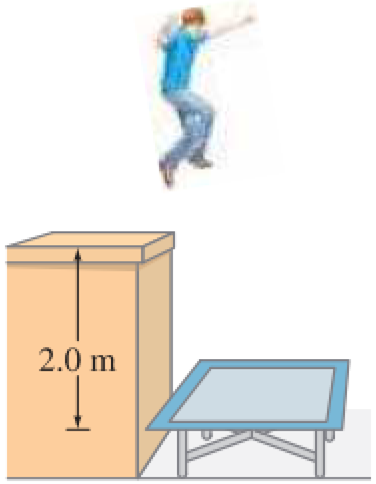


FIGURE 8–31
Problem 16.

16. (*a*) Since there are no dissipative forces present, the mechanical energy of the person–trampoline–Earth combination will be conserved. We take the level of the unstretched trampoline as the zero level for both elastic and gravitational potential energy. Call up the positive direction. Subscript 1 represents the jumper at the start of the jump, and subscript 2 represents the jumper upon arriving at the trampoline. There is no elastic potential energy involved in this part of the problem. We have $v_1 = 4.5\,\text{m/s}$, $y_1 = 2.0$ m, and $y_2 = 0$. Solve for v_2, the speed upon arriving at the trampoline.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + 0 \rightarrow$$

$$v_2 = \pm\sqrt{v_1^2 + 2gy_1} = \pm\sqrt{(4.5\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(2.0\,\text{m})} = \pm 7.710\,\text{m/s} \approx \boxed{7.7\,\text{m/s}}$$

The speed is the absolute value of v_2.

(*b*) Now let subscript 3 represent the jumper at the maximum stretch of the trampoline, and x represent the amount of stretch of the trampoline. We have $v_2 = -7.710\,\text{m/s}$, $y_2 = 0$, $x_2 = 0$, $v_3 = 0$, and $x_3 = y_3$. There is no elastic energy at position 2, but there is elastic energy at position 3. Also, the gravitational potential energy at position 3 is negative, and so $y_3 < 0$. A quadratic relationship results from the conservation of energy condition.

$$E_2 = E_3 \rightarrow \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \rightarrow$$

$$\tfrac{1}{2}mv_2^2 + 0 + 0 = 0 + mgy_3 + \tfrac{1}{2}ky_3^2 \rightarrow \tfrac{1}{2}ky_3^2 + mgy_3 - \tfrac{1}{2}mv_2^2 = 0 \rightarrow$$

$$y_3 = \frac{-mg \pm \sqrt{m^2g^2 - 4\left(\tfrac{1}{2}k\right)\left(-\tfrac{1}{2}mv_2^2\right)}}{2\left(\tfrac{1}{2}k\right)} = \frac{-mg \pm \sqrt{m^2g^2 + kmv_2^2}}{k}$$

$$= \frac{-(72\,\text{kg})(9.80\,\text{m/s}^2) \pm \sqrt{(72\,\text{kg})^2 (9.80\,\text{m/s}^2)^2 + (5.8\times10^4\ \text{N/m})(72\,\text{kg})(7.71\,\text{m/s})^2}}{(5.8\times10^4\ \text{N/m})}$$

$$= -0.284\,\text{m}\ ,\ 0.260\,\text{m}$$

Since $y_3 < 0$, $y_3 = \boxed{-0.28\,\text{m}}$..

The first term under the quadratic is about 500 times smaller than the second term, indicating that the problem could have been approximated by not even including gravitational potential energy for the final position. If that approximation were made, the result would have been found by taking the negative result from the following solution.

$$E_2 = E_3 \quad \rightarrow \quad \tfrac{1}{2} m v_2^2 = \tfrac{1}{2} k y_3^2 \quad \rightarrow \quad y_3 = v_2 \sqrt{\frac{m}{k}} = (7.71\,\text{m/s}) \sqrt{\frac{72\,\text{kg}}{5.8\times10^4\ \text{N/m}}} = \pm 0.27\,\text{m}$$

17. (II) The total energy E of an object of mass m that moves in one dimension under the influence of only conservative forces can be written as

$$E = \frac{1}{2} m v^2 + U.$$

Use conservation of energy, $dE/dt = 0$, to predict Newton's second law.

17. Take specific derivatives with respect to position, and note that E is constant.

$$E = \tfrac{1}{2} m v^2 + U \quad \rightarrow \quad \frac{dE}{dx} = \tfrac{1}{2} m \left(2v \frac{dv}{dx} \right) + \frac{dU}{dx} = mv \frac{dv}{dx} + \frac{dU}{dx} = 0$$

Use the chain rule to change $v \frac{dv}{dx}$ to $\frac{dx}{dt}\frac{dv}{dx} = \frac{dv}{dt}$.

$$mv \frac{dv}{dx} + \frac{dU}{dx} = 0 \quad \rightarrow \quad m \frac{dv}{dt} = -\frac{dU}{dx} \quad \rightarrow \quad \boxed{ma = F}$$

The last statement is Newton's second law.

20. (II) A roller-coaster car shown in Fig. 8–32 is pulled up to point 1 where it is released from rest. Assuming no friction, calculate the speed at points 2, 3, and 4.

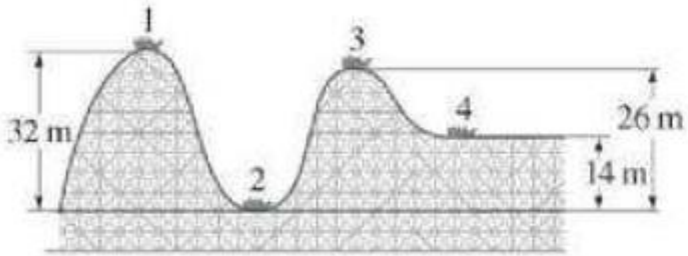


FIGURE 8–32 Problems 20 and 34.

23. (II) A block of mass m is attached to the end of a spring (spring stiffness constant k), Fig. 8–35. The mass is given an initial displacement x_0 from equilibrium, and an initial speed v_0. Ignoring friction and the mass of the spring, use energy methods to find (*a*) its maximum speed, and (*b*) its maximum stretch from equilibrium, in terms of the given quantities.

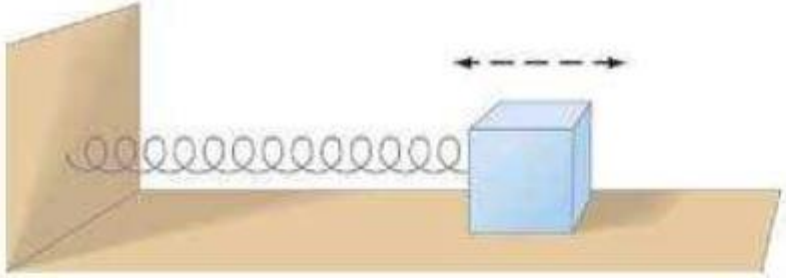


FIGURE 8–35 Problems 23, 37, and 38.

20. Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational potential energy. We have $v_1 = 0$ and $y_1 = 32\,\text{m}$.

Point 2: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \;;\; y_2 = 0 \rightarrow mgy_1 = \frac{1}{2}mv_2^2 \rightarrow$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\,\text{m/s}^2)(32\,\text{m})} = \boxed{25\,\text{m/s}}$$

Point 3: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_3^2 + mgy_3 \;;\; y_3 = 26\,\text{m} \rightarrow mgy_1 = \frac{1}{2}mv_3^2 + mgy_3 \rightarrow$

$$v_3 = \sqrt{2g(y_1 - y_3)} = \sqrt{2(9.80\,\text{m/s}^2)(6\,\text{m})} = \boxed{11\,\text{m/s}}$$

Point 4: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_4^2 + mgy_4 \;;\; y_4 = 14\,\text{m} \rightarrow mgy_1 = \frac{1}{2}mv_4^2 + mgy_1 \rightarrow$

$$v_4 = \sqrt{2g(y_1 - y_4)} = \sqrt{2(9.80\,\text{m/s}^2)(18\,\text{m})} = \boxed{19\,\text{m/s}}$$

23. At the release point the mass has both kinetic energy and elastic potential energy. The total energy is $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$. If friction is to be ignored, then that total energy is constant.

(*a*) The mass has its maximum speed at a displacement of 0, and so only has kinetic energy at that point.

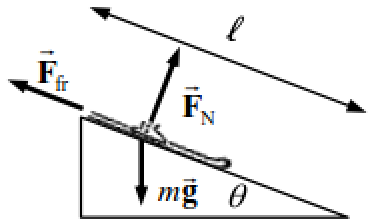
$$\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv_{\max}^2 \rightarrow v_{\max} = \boxed{\sqrt{v_0^2 + \frac{k}{m}x_0^2}}$$

(*b*) The mass has a speed of 0 at its maximum stretch from equilibrium, and so only has potential energy at that point.

$$\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kx_{\max}^2 \rightarrow x_{\max} = \boxed{\sqrt{x_0^2 + \frac{m}{k}v_0^2}}$$

31. (II) A ski starts from rest and slides down a 28° incline 85 m long. (*a*) If the coefficient of friction is 0.090, what is the ski's speed at the base of the incline? (*b*) If the snow is level at the foot of the incline and has the same coefficient of friction, how far will the ski travel along the level? Use energy methods.

31. (*a*) See the free-body diagram for the ski. Write Newton's second law for forces perpendicular to the direction of motion, noting that there is no acceleration perpendicular to the plane.



$$\sum F_\perp = F_N - mg\cos\theta \rightarrow F_N = mg\cos\theta \rightarrow$$

$$F_{fr} = \mu_k F_N = \mu_k mg\cos\theta$$

Now use conservation of energy, including the non-conservative friction force. Subscript 1 represents the ski at the top of the slope, and subscript 2 represents the ski at the bottom of the slope. The location of the ski at the bottom of the incline is the zero location for gravitational potential energy $(y=0)$. We have $v_1 = 0$, $y_1 = \ell\sin\theta$, and $y_2 = 0$. Write the conservation of energy condition, and solve for the final speed. Note that $F_{fr} = \mu_k F_N = \mu_k mg\cos\theta$.

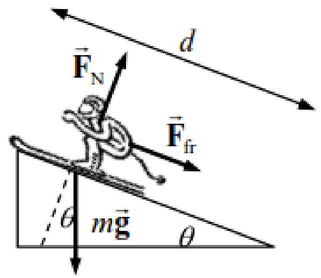
$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{fr}\ell \rightarrow mg\ell\sin\theta = \tfrac{1}{2}mv_2^2 + \mu_k mg\ell\cos\theta \rightarrow$$

$$v_2 = \sqrt{2g\ell(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80\,\text{m/s}^2)(85\,\text{m})(\sin 28^\circ - 0.090\cos 28^\circ)}$$

$$= 25.49\,\text{m/s} \approx \boxed{25\,\text{m/s}}$$

35. (II) A skier traveling 9.0 m/s reaches the foot of a steady upward 19° incline and glides 12 m up along this slope before coming to rest. What was the average coefficient of friction?

35. Consider the free-body diagram for the skier in the midst of the motion. Write Newton's second law for the direction perpendicular to the plane, with an acceleration of 0.



$$\sum F_\perp = F_N - mg\cos\theta = 0 \rightarrow F_N = mg\cos\theta \rightarrow$$

$$F_{fr} = \mu_k F_N = \mu_k mg\cos\theta$$

Apply conservation of energy to the skier, including the dissipative friction force. Subscript 1 represents the skier at the bottom of the slope, and subscript 2 represents the skier at the point furthest up the slope. The location of the skier at the bottom of the incline is the zero location for gravitational potential energy $(y=0)$. We have $v_1 = 9.0\,\text{m/s}$, $y_1 = 0$, $v_2 = 0$, and $y_2 = d\sin\theta$.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{fr}d \rightarrow \tfrac{1}{2}mv_1^2 + 0 = 0 + mgd\sin\theta + \mu_k mgd\cos\theta \rightarrow$$

$$\mu_k = \frac{\tfrac{1}{2}v_1^2 - gd\sin\theta}{gd\cos\theta} = \frac{v_1^2}{2gd\cos\theta} - \tan\theta = \frac{(9.0\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)(12\,\text{m})\cos 19^\circ} - \tan 19^\circ = \boxed{0.020}$$

62. (I) How long will it take a 1750-W motor to lift a 335-kg piano to a sixth-story window 16.0 m above?

63. (I) If a car generates 18 hp when traveling at a steady 95 km/h, what must be the average force exerted on the car due to friction and air resistance?

62. The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W = Fd\cos 0° = mgh$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P = \frac{W}{t} = \frac{mgh}{t} \rightarrow t = \frac{mgh}{P} = \frac{(335\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})}{1750\text{ W}} = \boxed{30.0\text{ s}}$$

63. The 18 hp is the power generated by the engine in creating a force on the ground to propel the car forward. The relationship between the power and the force is Eq. 8-21 with the force and velocity in the same direction, $P = Fv$. Thus the force to propel the car forward is found by $F = P/v$. If the car has a constant velocity, then the total resistive force must be of the same magnitude as the engine force, so that the net force is zero. Thus the total resistive force is also found by $F = P/v$.

$$F = \frac{P}{v} = \frac{(18\text{ hp})(746\text{ W/1 hp})}{(95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)} = \boxed{510\text{ N}}$$

83. A 62-kg skier starts from rest at the top of a ski jump, point A in Fig. 8–41, and travels down the ramp. If friction and air resistance can be neglected, (*a*) determine her speed v_B when she reaches the horizontal end of the ramp at B. (*b*) Determine the distance s to where she strikes the ground at C.

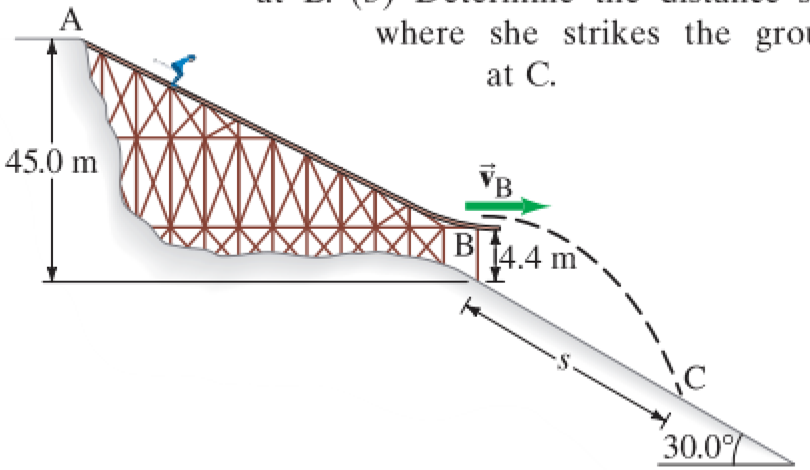


FIGURE 8–41 Problems 83 and 84.

83. (*a*) The speed v_B can be found from conservation of mechanical energy. Subscript A represents the skier at the top of the jump, and subscript B represents the skier at the end of the ramp. Point B is taken as the zero location for potential energy $(y = 0)$. We have $v_1 = 0$, $y_1 = 40.6\,\text{m}$, and $y_2 = 0$. Solve for v_2.

$$E_A = E_B \;\rightarrow\; \tfrac{1}{2}mv_A^2 + mgy_A = \tfrac{1}{2}mv_B^2 + mgy_B \;\rightarrow\; mgy_A = \tfrac{1}{2}mv_B^2 \;\rightarrow$$

$$v_B = \sqrt{2gy_A} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(40.6\,\text{m}\right)} = 28.209\,\text{m/s} \approx \boxed{28.2\,\text{m/s}}$$

(*b*) Now we use projectile motion. We take the origin of coordinates to be the point on the ground directly under the end of the ramp. Then an equation to describe the slope is $y_{\text{slope}} = -x\tan 30°$. The equations of projectile motion can be used to find an expression for the parabolic path that the skier follows after leaving the ramp. We take up to be the positive vertical direction. The initial y-velocity is 0, and the x-velocity is v_B as found above.

$$x = v_B t \;;\; y_{\text{proj}} = y_0 - \tfrac{1}{2}gt^2 = y_0 - \tfrac{1}{2}g\left(x/v_B\right)^2$$

The skier lands at the intersection of the two paths, so $y_{\text{slope}} = y_{\text{proj}}$.

$$y_{\text{slope}} = y_{\text{proj}} \;\rightarrow\; -x\tan 30° = y_0 - \tfrac{1}{2}g\left(\frac{x}{v_B}\right)^2 \;\rightarrow\; gx^2 - x\left(2v_B^2\tan 30°\right) - 2y_0v_B^2 = 0 \;\rightarrow$$

$$x = \frac{\left(2v_B^2\tan 30°\right) \pm \sqrt{\left(2v_B^2\tan 30°\right)^2 + 8gy_0v_B^2}}{2g} = \frac{\left(v_B^2\tan 30°\right) \pm \sqrt{\left(v_B^2\tan 30°\right)^2 + 2gy_0v_B^2}}{g}$$

Solving this with the given values gives $x = -7.09\,\text{m}, 100.8\,\text{m}$. The positive root is taken.

Finally, $s\cos 30.0° = x \;\rightarrow\; s = \dfrac{x}{\cos 30.0°} = \dfrac{100.8\,\text{m}}{\cos 30.0°} = \boxed{116\,\text{m}}$.

90. The small mass m sliding without friction along the looped track shown in Fig. 8–44 is to remain on the track at all times, even at the very top of the loop of radius r. (*a*) In terms of the given quantities, determine the minimum release height h. Next, if the actual release height is $2h$, calculate the normal force exerted (*b*) by the track at the bottom of the loop, (*c*) by the track at the top of the loop, and (*d*) by the track after the block exits the loop onto the flat section.

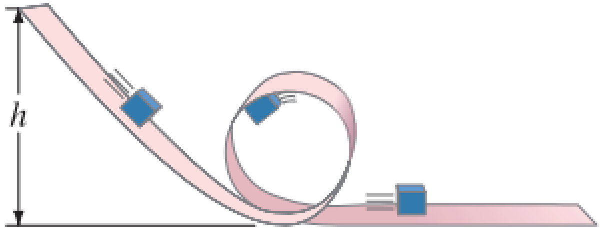
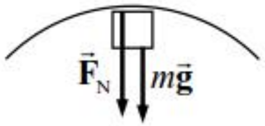


FIGURE 8–44
Problem 90.

90. (*a*) Draw a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's second law for the block, with down as positive. If the block is to be on the verge of falling off the track, then $F_N = 0$.



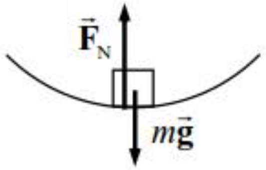
$$\sum F_R = F_N + mg = mv^2/r \rightarrow mg = mv_{top}^2/r \rightarrow v_{top} = \sqrt{gr}$$

Now use conservation of energy for the block. Since the track is frictionless, there are no non-conservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for potential energy $(y = 0)$. We have $v_1 = 0$, $y_1 = h$, $v_2 = \sqrt{gr}$, and $y_2 = 2r$. Solve for h.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow 0 + mgh = \tfrac{1}{2}mgr + 2mgr \rightarrow$$

$$h = \boxed{2.5r}$$

(*b*) See the free-body diagram for the block at the bottom of the loop. The net force is again centripetal, and must be upwards.



$$\sum F_R = F_N - mg = mv^2/r \rightarrow F_N = mg + mv_{bottom}^2/r$$

The speed at the bottom of the loop can be found from energy conservation, similar to what was done in part (*a*) above, by equating the energy at the release point (subscript 1) and the bottom of the loop (subscript 2). We now have $v_1 = 0$, $y_1 = 2h = 5r$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow 0 + 5mgr = \tfrac{1}{2}mv_{bottom}^2 + 0 \rightarrow$$

$$v_{bottom}^2 = 10gr \rightarrow F_N = mg + mv_{bottom}^2/r = mg + 10mg = \boxed{11mg}$$

(*c*) Again we use the free body diagram for the top of the loop, but now the normal force does not vanish. We again use energy conservation, with $v_1 = 0$, $y_1 = 3r$, and $y_2 = 0$. Solve for v_2.

$$\sum F_R = F_N + mg = mv^2/r \rightarrow F_N = mv_{top}^2/r - mg$$

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow 0 + 3mgr = \tfrac{1}{2}mv_{top}^2 + 0 \rightarrow$$

$$v_{top}^2 = 6gr \rightarrow F_N = mv_{top}^2/r - mg = 6mg - mg = \boxed{5mg}$$

(*d*) On the flat section, there is no centripetal force, and $F_N = \boxed{mg}$.