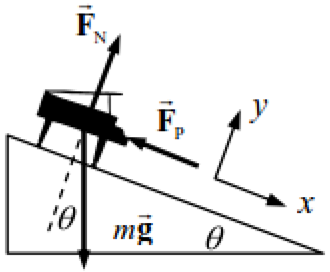
11. (II) A 380-kg piano slides 3.9 m down a 27° incline and is kept from accelerating by a man who is pushing back on it *parallel to the incline* (Fig. 7–21). Determine: (*a*) the force exerted by the man, (*b*) the work done by the man on the piano, (*c*) the work done by the force of gravity, and (*d*) the net work done on the piano. Ignore friction.

FIGURE 7–21
Problem 11.

11. The piano is moving with a constant velocity down the plane. $\vec{\mathbf{F}}_P$ is the force of the man pushing on the piano.



(*a*) Write Newton's second law on each direction for the piano, with an acceleration of 0.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad\rightarrow\quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_P = 0 \quad\rightarrow$$

$$F_P = mg\sin\theta = mg\sin\theta$$

$$= (380\,\text{kg})(9.80\,\text{m/s}^2)(\sin 27°) = 1691\,\text{N} \approx \boxed{1700\,\text{N}}$$

(*b*) The work done by the man is the work done by $\vec{\mathbf{F}}_P$. The angle between $\vec{\mathbf{F}}_P$ and the direction of motion is 180°. Use Eq. 7-1.

$$W_P = F_P d\cos 180° = -(1691\,\text{N})(3.9\,\text{m}) = -6595\,\text{J} \approx \boxed{-6600\,\text{J}}.$$

(*c*) The angle between the force of gravity and the direction of motion is 63°. Calculate the work done by gravity.

$$W_G = F_G d\cos 63° = mgd\cos 63° = (380\,\text{kg})(9.80\,\text{m/s}^2)(3.9\,\text{m})\cos 63°$$

$$= 6594\,\text{N} \approx \boxed{6600\,\text{J}}$$

(*d*) Since the piano is not accelerating, the net force on the piano is 0, and so the net work done on the piano is also 0. This can also be seen by adding the two work amounts calculated.

$$W_{\text{net}} = W_P + W_G = -6.6\times10^3\,\text{J} + 6.6\times10^3\,\text{J} = \boxed{0\,\text{J}}$$

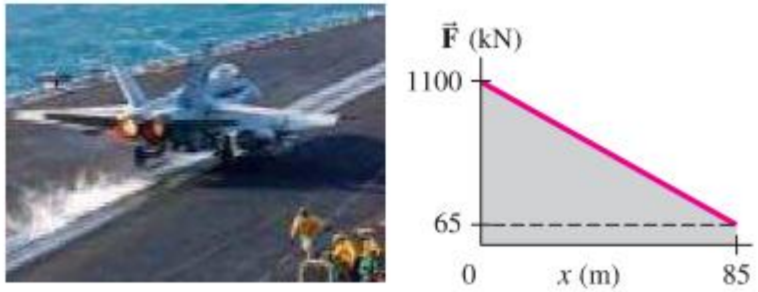
8. (II) Eight books, each 4.0 cm thick with mass 1.8 kg, lie flat on a table. How much work is required to stack them one on top of another?

8. The first book is already in position, so no work is required to position it. The second book must be moved upwards by a distance d, by a force equal to its weight, mg. The force and the displacement are in the same direction, so the work is mgd. The third book will need to be moved a distance of $2d$ by the same size force, so the work is $2mgd$. This continues through all seven books, with each needing to be raised by an additional amount of d by a force of mg. The total work done is

$$W = mgd + 2mgd + 3mgd + 4mgd + 5mgd + 6mgd + 7mgd$$

$$= 28mgd = 28(1.8\text{ kg})(9.8\text{ m/s}^2)(0.040\text{ m}) = \boxed{2.0\times10^1\text{ J}}$$

13. (II) A 17,000-kg jet takes off from an aircraft carrier via a catapult (Fig. 7–22a). The gases thrust out from the jet's engines exert a constant force of 130 kN on the jet; the force exerted on the jet by the catapult is plotted in Fig. 7–22b. Determine: (*a*) the work done on the jet by the gases expelled by its engines during launch of the jet; and (*b*) the work done on the jet by the catapult during launch of the jet.



(a) (b)

FIGURE 7–22 Problem 13.

13. (*a*) The gases exert a force on the jet in the same direction as the displacement of the jet. From the graph we see the displacement of the jet during launch is 85 m. Use Eq. 7-1 to find the work.

$$W_{gas} = F_{gas}d\cos 0° = (130\times10^3\text{N})(85\text{m}) = \boxed{1.1\times10^7\text{J}}$$

(*b*) The work done by catapult is the area underneath the graph in Figure 7-22. That area is a trapezoid.

$$W_{catapult} = \tfrac{1}{2}(1100\times10^3\text{N} + 65\times10^3\text{N})(85\text{m}) = \boxed{5.0\times10^7\text{J}}$$

16. (I) What is the dot product of $\vec{\mathbf{A}} = 2.0x^2\hat{\mathbf{i}} - 4.0x\hat{\mathbf{j}} + 5.0\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 11.0\hat{\mathbf{i}} + 2.5x\hat{\mathbf{j}}$?

17. (I) For any vector $\vec{\mathbf{V}} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}}$ show that

$$V_x = \hat{\mathbf{i}}\cdot\vec{\mathbf{V}}, \qquad V_y = \hat{\mathbf{j}}\cdot\vec{\mathbf{V}}, \qquad V_z = \hat{\mathbf{k}}\cdot\vec{\mathbf{V}}.$$

18. (I) Calculate the angle between the vectors:
$\vec{\mathbf{A}} = 6.8\hat{\mathbf{i}} - 3.4\hat{\mathbf{j}} - 6.2\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 8.2\hat{\mathbf{i}} + 2.3\hat{\mathbf{j}} - 7.0\hat{\mathbf{k}}$.

16. Use Eq. 7.4 to calculate the dot product.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z = (2.0x^2)(11.0) + (-4.0x)(2.5x) + (5.0)(0) = 22x^2 - 10x^2$$
$$= \boxed{12x^2}$$

17. Use Eq. 7.4 to calculate the dot product. Note that $\hat{\mathbf{i}} = 1\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, $\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, and $\hat{\mathbf{k}} = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 1\hat{\mathbf{k}}$.

$$\hat{\mathbf{i}}\bullet\vec{\mathbf{V}} = (1)V_x + (0)V_y + (0)V_z = V_x \qquad \hat{\mathbf{j}}\bullet\vec{\mathbf{V}} = (0)V_x + (1)V_y + (0)V_z = V_y$$
$$\hat{\mathbf{k}}\bullet\vec{\mathbf{V}} = (0)V_x + (0)V_y + (1)V_z = V_z$$

18. Use Eq. 7.4 and Eq. 7.2 to calculate the dot product, and then solve for the angle.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z = (6.8)(8.2) + (-3.4)(2.3) + (-6.2)(-7.0) = 91.34$$
$$A = \sqrt{(6.8^2) + (-3.4)^2 + (-6.2)^2} = 9.81 \qquad B = \sqrt{(8.2^2) + (2.3)^2 + (-7.0)^2} = 11.0$$
$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos\theta \rightarrow \theta = \cos^{-1}\frac{\vec{\mathbf{A}}\bullet\vec{\mathbf{B}}}{AB} = \cos^{-1}\frac{91.34}{(9.81)(11.0)} = \boxed{32^\circ}$$

30. (II) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are two vectors in the xy plane that make angles α and β with the x axis respectively. Evaluate the scalar product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and deduce the following trigonometric identity: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

31. (II) Suppose $\vec{\mathbf{A}} = 1.0\hat{\mathbf{i}} + 1.0\hat{\mathbf{j}} - 2.0\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = -1.0\hat{\mathbf{i}} + 1.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{k}}$, (*a*) what is the angle between these two vectors? (*b*) Explain the significance of the sign in part (*a*).

32. (II) Find a vector of unit length in the xy plane that is perpendicular to $3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}$.

30. We can represent the vectors as $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = A\cos\alpha\hat{\mathbf{i}} + A\sin\alpha\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}$ $= B\cos\beta\hat{\mathbf{i}} + B\sin\beta\hat{\mathbf{j}}$. The angle between the two vectors is $\alpha - \beta$. Use Eqs. 7-2 and 7-4 to express the dot product.

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos(\alpha-\beta) = A_xB_x + A_yB_y = A\cos\alpha B\cos\beta + A\sin\alpha B\sin\beta \rightarrow$$

$$AB\cos(\alpha-\beta) = AB\cos\alpha\cos\beta + AB\sin\alpha\sin\beta \rightarrow \boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

31. (*a*) Use the two expressions for dot product, Eqs. 7-2 and 7-4, to find the angle between the two vectors.

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z \rightarrow$$

$$\theta = \cos^{-1}\frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$$= \cos^{-1}\frac{(1.0)(-1.0)+(1.0)(1.0)+(-2.0)(2.0)}{\left[(1.0)^2+(1.0)^2+(-2.0)^2\right]^{1/2}\left[(-1.0)^2+(1.0)^2+(2.0)^2\right]^{1/2}}$$

$$= \cos^{-1}\left(-\tfrac{2}{3}\right) = 132^\circ \approx \boxed{130^\circ}$$

(*b*) The negative sign in the argument of the inverse cosine means that the angle between the two vectors is obtuse.

32. To be perpendicular to the given vector means that the dot product will be 0. Let the unknown vector be given as $\hat{\mathbf{u}} = u_x\hat{\mathbf{i}} + u_y\hat{\mathbf{j}}$.

$$\hat{\mathbf{u}}\cdot(3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}) = 3.0u_x + 4.0u_y \rightarrow u_y = -0.75u_x \text{ ; unit length} \rightarrow u_x^2 + u_y^2 = 1 \rightarrow$$

$$u_x^2 + u_y^2 = u_x^2 + (-0.75u_x)^2 = 1.5625u_x^2 = 1 \rightarrow u_x = \pm\frac{1}{\sqrt{1.5625}} = \pm 0.8 \text{ , } u_y = \mp 0.6$$

So the two possible vectors are $\boxed{\hat{\mathbf{u}} = 0.8\hat{\mathbf{i}} - 0.6\hat{\mathbf{j}}}$ and $\boxed{\hat{\mathbf{u}} = -0.8\hat{\mathbf{i}} + 0.6\hat{\mathbf{j}}}$.

Note that it is very easy to get a non-unit vector perpendicular to another vector in two dimensions, simply by interchanging the coordinates and negating one of them. So a non-unit vector perpendicular to $(3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ could be either $(4.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}})$ or $(-4.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$. Then divide each of those vectors by its magnitude (5.0) to get the possible unit vectors.

38. (II) If it requires 5.0 J of work to stretch a particular spring by 2.0 cm from its equilibrium length, how much more work will be required to stretch it an additional 4.0 cm?

38. The work required to stretch a spring from equilibrium is proportional to the length of stretch, squared. So if we stretch the spring to 3 times its original distance, a total of 9 times as much work is required for the total stretch. Thus it would take 45.0 J to stretch the spring to a total of 6.0 cm. Since 5.0 J of work was done to stretch the first 2.0 cm, $\boxed{40.0\ \text{J}}$ of work is required to stretch it the additional 4.0 cm.

This could also be done by calculating the spring constant from the data for the 2.0 cm stretch, and then using that spring constant to find the work done in stretching the extra distance.

47. (II) An object, moving along the circumference of a circle with radius R, is acted upon by a force of constant magnitude F. The force is directed at all times at a 30° angle with respect to the tangent to the circle as shown in Fig. 7–25. Determine the work done by this force when the object moves along the half circle from A to B.

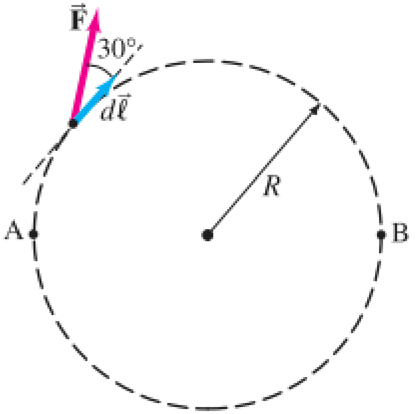


FIGURE 7–25 Problem 47.

47. Since the force is of constant magnitude and always directed at $30°$ to the displacement, we have a simple expression for the work done as the object moves.

$$W = \int_{\text{start}}^{\text{finish}} \vec{\mathbf{F}} \cdot d\vec{\ell} = \int_{\text{start}}^{\text{finish}} F \cos 30° d\ell = F \cos 30° \int_{\text{start}}^{\text{finish}} d\ell = F \cos 30° \pi R = \boxed{\frac{\sqrt{3}\pi FR}{2}}$$

50. (I) At room temperature, an oxygen molecule, with mass of 5.31×10^{-26} kg, typically has a kinetic energy of about 6.21×10^{-21} J. How fast is it moving?

50. Find the velocity from the kinetic energy, using Eq. 7-10.

$$K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(6.21\times10^{-21}\,\text{J}\right)}{5.31\times10^{-26}}} = \boxed{484\,\text{m/s}}$$

57. (II) A mass m is attached to a spring which is held stretched a distance x by a force F (Fig. 7–28), and then released. The spring compresses, pulling the mass. Assuming there is no friction, determine the speed of the mass m when the spring returns: (*a*) to its normal length $(x = 0)$; (*b*) to half its original extension $(x/2)$.

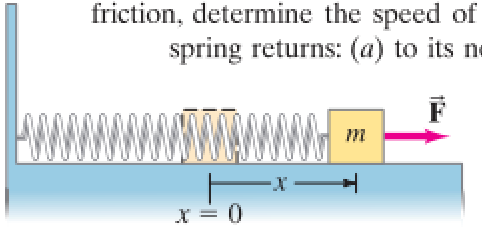


FIGURE 7–28
Problem 57.

57. (*a*) The spring constant is found by the magnitudes of the initial force and displacement, and so $k = F/x$. As the spring compresses, it will do the same amount of work on the block as was done on the spring to stretch it. The work done is positive because the force of the spring is parallel to the displacement of the block. Use the work-energy theorem to determine the speed of the block.

$$W_{\substack{\text{on block}\\\text{during}\\\text{compression}}} = \Delta K_{\text{block}} = W_{\substack{\text{on spring}\\\text{during}\\\text{stretching}}} \rightarrow \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}kx^2 = \tfrac{1}{2}\frac{F}{x}x^2 \rightarrow v_f = \boxed{\sqrt{\frac{Fx}{m}}}$$

(*b*) Now we must find how much work was done on the spring to stretch it from $x/2$ to x. This will be the work done on the block as the spring pulls it back from x to $x/2$.

$$W_{\substack{\text{on spring}\\\text{during}\\\text{stretching}}} = \int_{x/2}^{x} F\,dx = \int_{x/2}^{x} kx\,dx = \tfrac{1}{2}kx^2\Big|_{x/2}^{x} = \tfrac{1}{2}kx^2 - \tfrac{1}{2}k\left(x/2\right)^2 = \tfrac{3}{8}kx^2$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{3}{8}kx^2 \rightarrow v_f = \boxed{\sqrt{\frac{3Fx}{4m}}}$$

63. (II) (*a*) How much work is done by the horizontal force $F_P = 150\,\text{N}$ on the 18-kg block of Fig. 7–29 when the force pushes the block 5.0 m up along the 32° frictionless incline? (*b*) How much work is done by the gravitational force on the block during this displacement? (*c*) How much work is done by the normal force? (*d*) What is the speed of the block (assume that it is zero initially) after this displacement? [*Hint*: Work-energy involves *net* work done.]

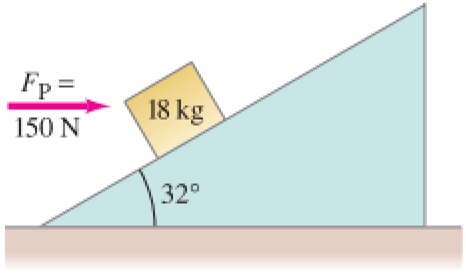


FIGURE 7–29
Problems 63 and 64.

63. (*a*) The angle between the pushing force and the displacement is 32°.

$$W_P = F_P d\cos\theta = (150\,\text{N})(5.0\,\text{m})\cos 32° = 636.0\,\text{J} \approx \boxed{640\,\text{J}}$$

(*b*) The angle between the force of gravity and the displacement is 122°.

$$W_G = F_G d\cos\theta = mgd\cos\theta = (18\,\text{kg})(9.80\,\text{m/s}^2)(5.0\,\text{m})\cos 122° = -467.4\,\text{J} \approx \boxed{-470\,\text{J}}$$

(*c*) Because the normal force is perpendicular to the displacement, the work done by the normal force is $\boxed{0}$.

(*d*) The net work done is the change in kinetic energy.

$$W = W_P + W_g + W_N = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \rightarrow$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(636.0\,\text{J} - 467.4\,\text{J})}{(18\,\text{kg})}} = \boxed{4.3\,\text{m/s}}$$

75. Two forces, $\vec{\mathbf{F}}_1 = (1.50\hat{\mathbf{i}} - 0.80\hat{\mathbf{j}} + 0.70\hat{\mathbf{k}})\,\text{N}$ and $\vec{\mathbf{F}}_2 = (-0.70\hat{\mathbf{i}} + 1.20\hat{\mathbf{j}})\,\text{N}$, are applied on a moving object of mass 0.20 kg. The displacement vector produced by the two forces is $\vec{\mathbf{d}} = (8.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}} + 5.0\hat{\mathbf{k}})\,\text{m}$. What is the work done by the two forces?

75. Since the forces are constant, we may use Eq. 7-3 to calculate the work done.

$$W_{\text{net}} = (\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2)\cdot\vec{\mathbf{d}} = \left[(1.50\hat{\mathbf{i}} - 0.80\hat{\mathbf{j}} + 0.70\hat{\mathbf{k}})\,\text{N} + (-0.70\hat{\mathbf{i}} + 1.20\hat{\mathbf{j}})\,\text{N}\right]\cdot\left[(8.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}} + 5.0\hat{\mathbf{k}})\,\text{m}\right]$$

$$= \left[(0.80\hat{\mathbf{i}} + 0.40\hat{\mathbf{j}} + 0.70\hat{\mathbf{k}})\,\text{N}\right]\cdot\left[(8.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}} + 5.0\hat{\mathbf{k}})\,\text{m}\right] = (6.4 + 2.4 + 3.5)\,\text{J} = \boxed{12.3\,\text{J}}$$

86. A simple pendulum consists of a small object of mass m (the "bob") suspended by a cord of length ℓ (Fig. 7–32) of negligible mass. A force $\vec{\mathbf{F}}$ is applied in the horizontal direction (so $\vec{\mathbf{F}} = F\hat{\mathbf{i}}$), moving the bob very slowly so the acceleration is essentially zero. (Note that the magnitude of $\vec{\mathbf{F}}$ will need to vary with the angle θ that the cord makes with the vertical at any moment.) (*a*) Determine the work done by this force, $\vec{\mathbf{F}}$, to move the pendulum from $\theta = 0$ to $\theta = \theta_0$. (*b*) Determine the work done by the gravitational force on the bob, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$, and the work done by the force $\vec{\mathbf{F}}_T$ that the cord exerts on the bob.

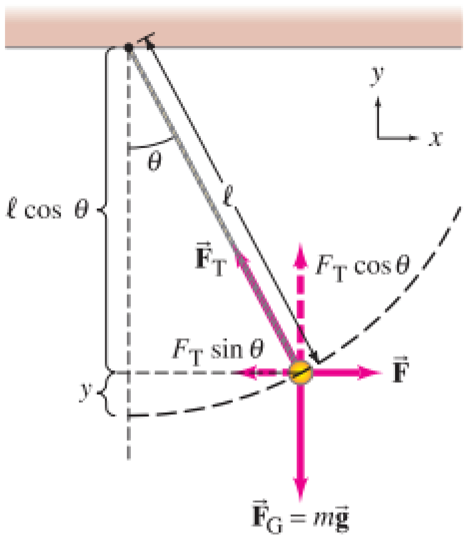


FIGURE 7–32
Problem 86.

86. Because the acceleration is essentially 0, the net force on the mass is 0. The magnitude of $\vec{\mathbf{F}}$ is found with the help of the free-body diagram in the textbook.

$$\sum F_y = F_T \cos\theta - mg = 0 \rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\sum F_x = F - F_T \sin\theta = 0 \rightarrow F = F_T \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

(*a*) A small displacement of the object along the circular path is given by $dr = \ell d\theta$, based on the definition of radian measure. The force $\vec{\mathbf{F}}$ is at an angle θ to the direction of motion. We use the symbol $d\vec{\mathbf{r}}$ for the infinitesimal displacement, since the symbol ℓ is already in use as the length of the pendulum.

$$W_F = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{\theta=0}^{\theta=\theta_0} F\cos\theta \ell d\theta = \ell \int_{\theta=0}^{\theta=\theta_0} (mg\tan\theta)\cos\theta d\theta = mg\ell \int_{\theta=0}^{\theta=\theta_0} \sin\theta d\theta$$

$$= -mg\ell\cos\theta\Big|_0^{\theta_0} = \boxed{mg\ell(1-\cos\theta_0)}$$

(*b*) The angle between $m\vec{\mathbf{g}}$ and the direction of motion is $(90+\theta)$.

$$W_G = \int m\vec{\mathbf{g}} \cdot d\vec{\mathbf{r}} = mg\ell \int_{\theta=0}^{\theta=\theta_0} \cos(90^\circ+\theta)d\theta = -mg\ell \int_{\theta=0}^{\theta=\theta_0} \sin\theta d\theta$$

$$= mg\ell\cos\theta\Big|_0^{\theta_0} = \boxed{mg\ell(\cos\theta_0 - 1)}$$

Alternatively, it is proven in problem 36 that the shape of the path does not determine the work done by gravity – only the height change. Since this object is rising, gravity will do negative work.

$$W_G = mgd\cos\phi = mg(\text{height})\cos 180^\circ = -mgy_{\text{final}} = -mg(\ell - \ell\cos\theta_0)$$

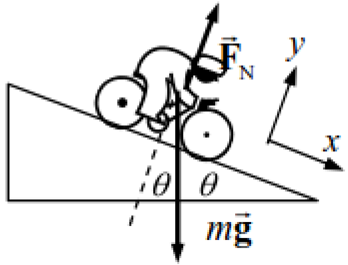
$$= \boxed{mg\ell(\cos\theta_0 - 1)}$$

Since $\vec{\mathbf{F}}_T$ is perpendicular to the direction of motion, it does $\boxed{0}$ work on the bob.

Note that the total work done is 0, since the object's kinetic energy does not change.

89. A cyclist starts from rest and coasts down a 4.0° hill. The mass of the cyclist plus bicycle is 85 kg. After the cyclist has traveled 250 m, (*a*) what was the net work done by gravity on the cyclist? (*b*) How fast is the cyclist going? Ignore air resistance.

89. (*a*) The work done by gravity is given by Eq. 7-1.



$$W_G = mgd\cos(90-\theta) = (85\,\text{kg})(9.80\,\text{m/s}^2)(250\,\text{m})\cos 86.0^\circ$$

$$= 1.453\times10^4\,\text{J} \approx \boxed{1.5\times10^4\,\text{J}}$$

(*b*) The work is the change in kinetic energy. The initial kinetic energy is 0.

$$W_G = \Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 \;\rightarrow\; v_f = \sqrt{\frac{2W_G}{m}} = \sqrt{\frac{2(1.453\times10^4\,\text{J})}{85\,\text{kg}}} = \boxed{18\,\text{m/s}}$$

91. A small mass m hangs at rest from a vertical rope of length ℓ that is fixed to the ceiling. A force $\vec{F}$ then pushes on the mass, perpendicular to the taut rope at all times, until the rope is oriented at an angle $\theta = \theta_0$ and the mass has been raised by a vertical distance h (Fig. 7–34). Assume the force's magnitude F is adjusted so that the mass moves at constant speed along its curved trajectory. Show that the work done by $\vec{F}$ during this process equals mgh, which is equivalent to the amount of work it takes to slowly lift a mass m straight up by a height h. [*Hint*: When the angle is increased by $d\theta$ (in radians), the mass moves along an arc length $ds = \ell\, d\theta$.]

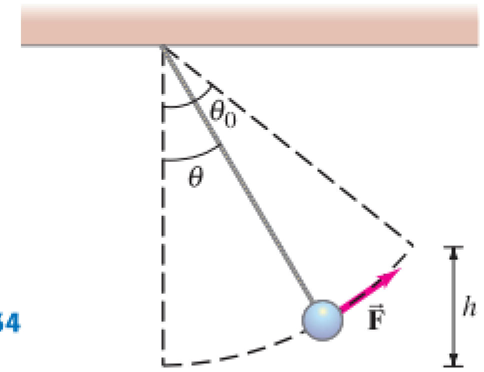
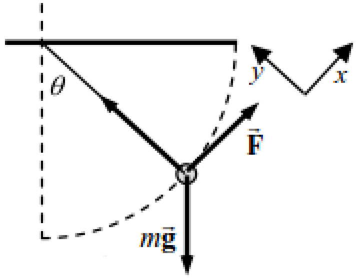


FIGURE 7–34
Problem 91.

91. Refer to the free body diagram. The coordinates are defined simply to help analyze the components of the force. At any angle θ, since the mass is not accelerating, we have the following.



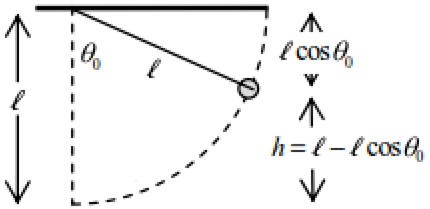
$$\sum F_x = F - mg\sin\theta = 0 \;\rightarrow\; F = mg\sin\theta$$

Find the work done in moving the mass from $\theta = 0$ to $\theta = \theta_0$.

$$W_F = \int \vec{F}\cdot d\vec{s} = \int_{\theta=0}^{\theta=\theta_0} F\cos 0^\circ\,\ell\,d\theta = mg\ell\int_{\theta=0}^{\theta=\theta_0}\sin\theta\,d\theta$$

$$= -mg\ell\cos\theta\Big|_0^{\theta_0} = mg\ell(1-\cos\theta_0)$$

See the second diagram to find the height that the mass has risen. We see that $h = \ell - \ell\cos\theta_0 = \ell(1-\cos\theta_0)$, and so



$$\boxed{W_F = mg\ell(1-\cos\theta_0) = mgh}.$$