6. If $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, is V necessarily greater than V_1 and/or V_2? Discuss.

7. Two vectors have length $V_1 = 3.5$ km and $V_2 = 4.0$ km. What are the maximum and minimum magnitudes of their vector sum?

8. Can two vectors, of unequal magnitude, add up to give the zero vector? Can *three* unequal vectors? Under what conditions?

9. Can the magnitude of a vector ever (*a*) equal, or (*b*) be less than, one of its components?

10. Can a particle with constant speed be accelerating? What if it has constant velocity?

11. Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?

6. V is the magnitude of the vector $\vec{\mathbf{V}}$; it is not necessarily larger than the magnitudes V_1 and V_2. For instance, if $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ have the same magnitude as each other and are in opposite directions, then V is zero.

7. The maximum magnitude of the sum is 7.5 km, in the case where the vectors are parallel. The minimum magnitude of the sum is 0.5 km, in the case where the vectors are antiparallel.

8. No. The only way that two vectors can add up to give the zero vector is if they have the same magnitude and point in exactly opposite directions. However, three vectors of unequal magnitudes can add up to the zero vector. As a one-dimensional example, a vector 10 units long in the positive x direction added to two vectors of 4 and 6 units each in the negative x direction will result in the zero vector. In two dimensions, consider any three vectors that when added form a triangle.

9. (*a*) Yes. In three dimensions, the magnitude of a vector is the square root of the sum of the squares of the components. If two of the components are zero, the magnitude of the vector is equal to the magnitude of the remaining component.
(*b*) No.

10. Yes. A particle traveling around a curve while maintaining a constant speed is accelerating because its direction is changing. A particle with a constant velocity cannot be accelerating, since the velocity is not changing in magnitude or direction.

11. The odometer and the speedometer of the car both measure scalar quantities (distance and speed, respectively).

21. If you stand motionless under an umbrella in a rainstorm where the drops fall vertically you remain relatively dry. However, if you start running, the rain begins to hit your legs even if they remain under the umbrella. Why?

21. As you run forward, the umbrella also moves forward and stops raindrops that are at its height above the ground. Raindrops that have already passed the height of the umbrella continue to move toward the ground unimpeded. As you run, you move into the space where the raindrops are continuing to fall (below the umbrella). Some of them will hit your legs and you will get wet.

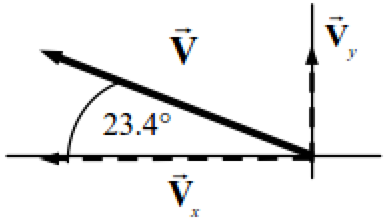
5. (II) $\vec{\mathbf{V}}$ is a vector 24.8 units in magnitude and points at an angle of 23.4° above the negative x axis. (*a*) Sketch this vector. (*b*) Calculate V_x and V_y. (*c*) Use V_x and V_y to obtain (again) the magnitude and direction of $\vec{\mathbf{V}}$. [*Note*: Part (*c*) is a good way to check if you've resolved your vector correctly.]

5. (*a*) See the accompanying diagram

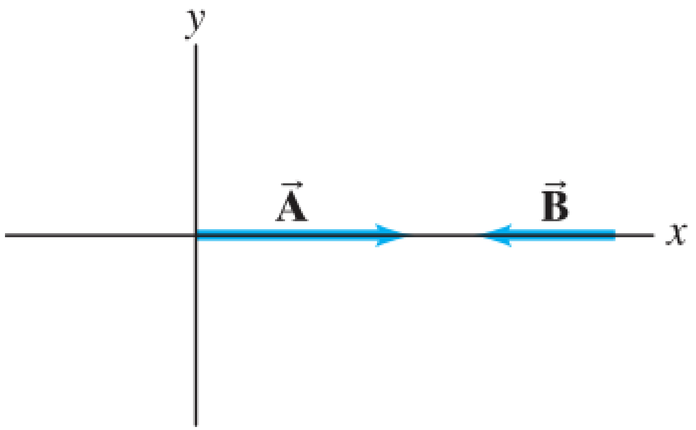
(*b*) $V_x = -24.8\cos 23.4° = \boxed{-22.8 \text{ units}}$ $V_y = 24.8\sin 23.4° = \boxed{9.85 \text{ units}}$

(*c*) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-22.8)^2 + (9.85)^2} = \boxed{24.8 \text{ units}}$

$\theta = \tan^{-1}\dfrac{9.85}{22.8} = \boxed{23.4° \text{above the } -x \text{ axis}}$



6. (II) Figure 3–36 shows two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, whose magnitudes are $A = 6.8$ units and $B = 5.5$ units. Determine $\vec{\mathbf{C}}$ if (*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, (*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, (*c*) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}}$. Give the magnitude and direction for each.



6. We see from the diagram that $\vec{\mathbf{A}} = 6.8\hat{\mathbf{i}}$ and $\vec{\mathbf{B}} = -5.5\hat{\mathbf{i}}$.

(*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.8\hat{\mathbf{i}} + (-5.5)\hat{\mathbf{i}} = \boxed{1.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{1.3 \text{ units}}$, and the direction is $\boxed{+x}$.

(*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 6.8\hat{\mathbf{i}} - (-5.5)\hat{\mathbf{i}} = \boxed{12.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{12.3 \text{ units}}$, and the direction is $\boxed{+x}$.

(*c*) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = (-5.5)\hat{\mathbf{i}} - 6.8\hat{\mathbf{i}} = \boxed{-12.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{12.3 \text{ units}}$, and the direction is $\boxed{-x}$.

8. (II) Let $\vec{\mathbf{V}}_1 = -6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}$ and $\vec{\mathbf{V}}_2 = 4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}$. Determine the magnitude and direction of (*a*) $\vec{\mathbf{V}}_1$, (*b*) $\vec{\mathbf{V}}_2$, (*c*) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ and (*d*) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$.

8. (*a*) $\vec{\mathbf{V}}_1 = -6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}} \quad V_1 = \sqrt{6.0^2 + 8.0^2} = \boxed{10.0} \quad \theta = \tan^{-1}\frac{8.0}{-6.0} = \boxed{127^\circ}$

(*b*) $\vec{\mathbf{V}}_2 = 4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}} \quad V_2 = \sqrt{4.5^2 + 5.0^2} = \boxed{6.7} \quad \theta = \tan^{-1}\frac{-5.0}{4.5} = \boxed{312^\circ}$

(*c*) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = (-6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}) + (4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}) = -1.5\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}$

$\left|\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\right| = \sqrt{1.5^2 + 3.0^2} = \boxed{3.4} \quad \theta = \tan^{-1}\frac{3.0}{-1.5} = \boxed{117^\circ}$

(*d*) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = (4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}) - (-6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}) = 10.5\hat{\mathbf{i}} - 13.0\hat{\mathbf{j}}$

$\left|\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1\right| = \sqrt{10.5^2 + 13.0^2} = \boxed{16.7} \quad \theta = \tan^{-1}\frac{-13.0}{10.5} = \boxed{309^\circ}$

10. (II) Three vectors are shown in Fig. 3–38. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (*a*) components, (*b*) magnitude and angle with *x* axis.

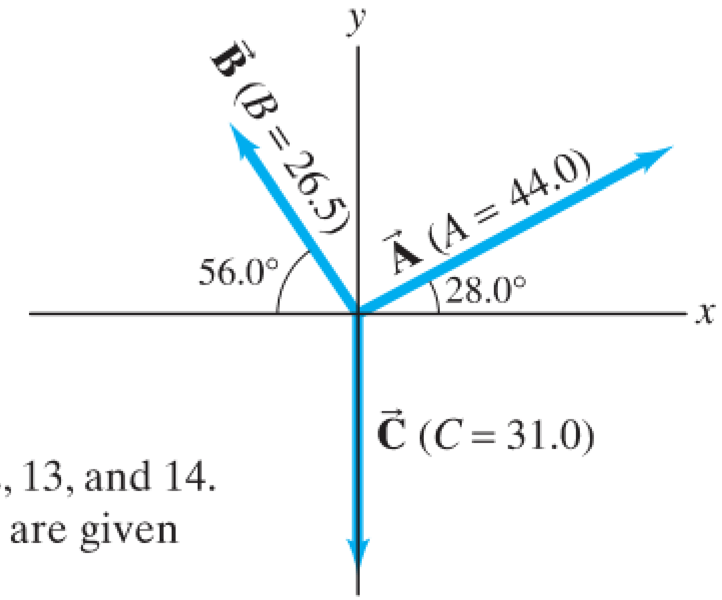


FIGURE 3–38
Problems 10, 11, 12, 13, and 14. Vector magnitudes are given in arbitrary units.

10. $A_x = 44.0\cos 28.0° = 38.85$ $\quad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82$ $\quad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270° = 0.0$ $\quad C_y = 31.0\sin 270° = -31.0$

(*a*) $(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}})_x = 38.85 + (-14.82) + 0.0 = 24.03 = \boxed{24.0}$

$(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}})_y = 20.66 + 21.97 + (-31.0) = 11.63 = \boxed{11.6}$

(*b*) $|\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}| = \sqrt{(24.03)^2 + (11.63)^2} = \boxed{26.7}$ $\qquad \theta = \tan^{-1}\frac{11.63}{24.03} = \boxed{25.8°}$

11. (II) (*a*) Given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Fig. 3–38, determine $\vec{\mathbf{B}} - \vec{\mathbf{A}}$. (*b*) Determine $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ without using your answer in (*a*). Then compare your results and see if they are opposite.

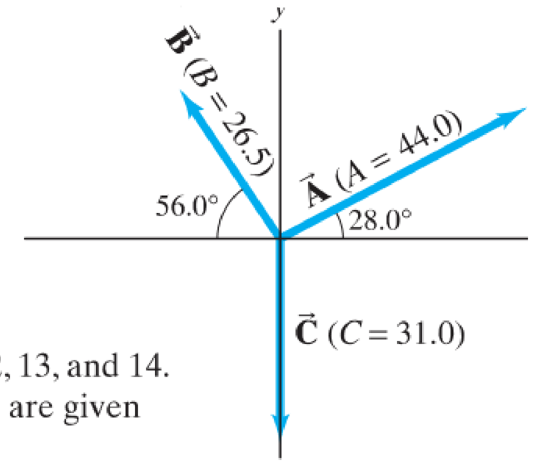


FIGURE 3–38
Problems 10, 11, 12, 13, and 14. Vector magnitudes are given in arbitrary units.

11. $A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82 \qquad B_y = 26.5\sin 56.0° = 21.97$

(*a*) $\left(\vec{\mathbf{B}} - \vec{\mathbf{A}}\right)_x = (-14.82) - 38.85 = -53.67 \qquad \left(\vec{\mathbf{B}} - \vec{\mathbf{A}}\right)_y = 21.97 - 20.66 = 1.31$

Note that since the *x* component is negative and the *y* component is positive, the vector is in the 2nd quadrant.

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \boxed{-53.7\hat{\mathbf{i}} + 1.31\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{B}} - \vec{\mathbf{A}}\right| = \sqrt{(-53.67)^2 + (1.31)^2} = \boxed{53.7} \qquad \theta_{B-A} = \tan^{-1}\frac{1.31}{-53.67} = \boxed{1.4° \text{ above } -x \text{ axis}}$$

(*b*) $\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right)_x = 38.85 - (-14.82) = 53.67 \qquad \left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right)_y = 20.66 - 21.97 = -1.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{53.7\hat{\mathbf{i}} - 1.31\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{A}} - \vec{\mathbf{B}}\right| = \sqrt{(53.67)^2 + (-1.31)^2} = \boxed{53.7} \qquad \theta = \tan^{-1}\frac{-1.31}{53.7} = \boxed{1.4° \text{ below } + x \text{ axis}}$$

Comparing the results shows that $\vec{\mathbf{B}} - \vec{\mathbf{A}} = -\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right)$.

17. (I) The position of a particular particle as a function of time is given by $\vec{\mathbf{r}} = (9.60\,t\,\hat{\mathbf{i}} + 8.85\hat{\mathbf{j}} - 1.00\,t^2\hat{\mathbf{k}})\,\mathrm{m}$. Determine the particle's velocity and acceleration as a function of time.

17. Differentiate the position vector in order to determine the velocity, and differentiate the velocity in order to determine the acceleration.

$$\vec{\mathbf{r}} = \left(9.60t\,\hat{\mathbf{i}} + 8.85\,\hat{\mathbf{j}} - 1.00t^2\hat{\mathbf{k}}\right)\mathrm{m} \;\rightarrow\; \vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \boxed{\left(9.60\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{k}}\right)\mathrm{m/s}} \;\rightarrow$$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \boxed{-2.00\,\hat{\mathbf{k}}\ \mathrm{m/s^2}}$$

22. (II) (*a*) A skier is accelerating down a 30.0° hill at 1.80 m/s (Fig. 3–39). What is the vertical component of her accelera tion? (*b*) How long will it take her to reach the bottom c the hill, assuming she starts from rest and accelerate uniformly, if the elevation change is 325 m?

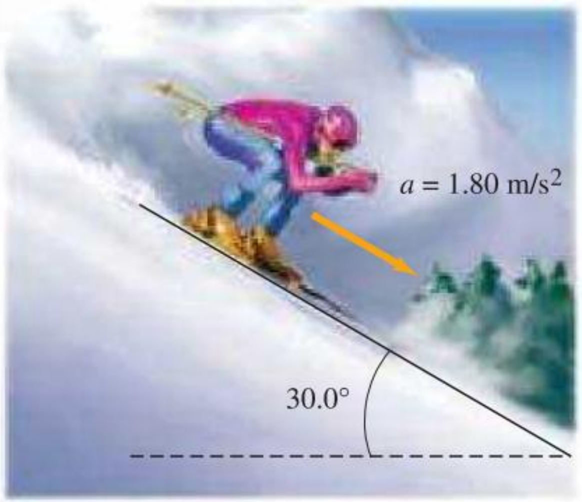


FIGURE 3–39 Problem 22.

22. Choose downward to be the positive *y* direction for this problem. Her acceleration is directed along the slope.

(*a*) The vertical component of her acceleration is directed downward, and its magnitude will be given by $a_y = a\sin\theta = \left(1.80\,\mathrm{m/s^2}\right)\sin 30.0^\circ = \boxed{0.900\,\mathrm{m/s^2}}$.

(*b*) The time to reach the bottom of the hill is calculated from Eq. 2-12b, with a *y* displacement of 325 m, $v_{y0} = 0$, and $a_y = 0.900\,\mathrm{m/s^2}$.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 325\ \mathrm{m} = 0 + 0 + \tfrac{1}{2}\left(0.900\,\mathrm{m/s^2}\right)(t)^2 \;\rightarrow$$

$$t = \sqrt{\frac{2(325\ \mathrm{m})}{\left(0.900\,\mathrm{m/s^2}\right)}} = \boxed{26.9\ \mathrm{s}}$$

26. (II) An object, which is at the origin at time $t = 0$, has initial velocity $\vec{\mathbf{v}}_0 = (-14.0\hat{\mathbf{i}} - 7.0\hat{\mathbf{j}})\,\mathrm{m/s}$ and constant acceleration $\vec{\mathbf{a}} = (6.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\,\mathrm{m/s^2}$. Find the position $\vec{\mathbf{r}}$ where the object comes to rest (momentarily).

26. The position vector can be found from Eq. 3-13b, since the acceleration vector is constant. The time at which the object comes to rest is found by setting the velocity vector equal to 0. Both components of the velocity must be 0 at the same time for the object to be at rest.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = \left(-14\hat{\mathbf{i}} - 7.0\hat{\mathbf{j}}\right)\mathrm{m/s} + \left(6.0t\,\hat{\mathbf{i}} + 3.0t\,\hat{\mathbf{j}}\right)\mathrm{m/s} = \left[(-14 + 6.0t)\,\hat{\mathbf{i}} + (-7.0 + 3.0t)\,\hat{\mathbf{j}}\right]\mathrm{m/s}$$

$$\vec{\mathbf{v}}_{\text{rest}} = \left(0.0\,\hat{\mathbf{i}} + 0.0\hat{\mathbf{j}}\right)\mathrm{m/s} = \left[(-14 + 6.0t)\,\hat{\mathbf{i}} + (-7.0 + 3.0t)\,\hat{\mathbf{j}}\right]\mathrm{m/s} \;\rightarrow$$

$$(v_x)_{\text{rest}} = 0.0 = -14 + 6.0t \;\rightarrow\; t = \frac{14}{6.0}\,\mathrm{s} = \tfrac{7}{3}\,\mathrm{s}$$

$$(v_y)_{\text{rest}} = 0.0 = -7.0 + 3.0t \;\rightarrow\; t = \frac{7.0}{3.0}\,\mathrm{s} = \tfrac{7}{3}\,\mathrm{s}$$

Since both components of velocity are 0 at $t = \tfrac{7}{3}\,\mathrm{s}$, the object is at rest at that time.

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 = \left(0.0\,\hat{\mathbf{i}} + 0.0\hat{\mathbf{j}}\right)\mathrm{m} + \left(-14t\,\hat{\mathbf{i}} - 7.0t\,\hat{\mathbf{j}}\right)\mathrm{m} + \tfrac{1}{2}\left(6.0t^2\hat{\mathbf{i}} + 3.0t^2\hat{\mathbf{j}}\right)\mathrm{m}$$

$$= \left(-14\left(\tfrac{7}{3}\right)\hat{\mathbf{i}} - 7.0\left(\tfrac{7}{3}\right)\hat{\mathbf{j}}\right)\mathrm{m} + \tfrac{1}{2}\left(6.0\left(\tfrac{7}{3}\right)^2\hat{\mathbf{i}} + 3.0\left(\tfrac{7}{3}\right)^2\hat{\mathbf{j}}\right)\mathrm{m}$$

$$= \left(-14\left(\tfrac{7}{3}\right) + \tfrac{1}{2}6.0\left(\tfrac{7}{3}\right)^2\right)\hat{\mathbf{i}}\,\mathrm{m} + \left(-7.0\left(\tfrac{7}{3}\right) + \tfrac{1}{2}3.0\left(\tfrac{7}{3}\right)^2\right)\hat{\mathbf{j}}\,\mathrm{m}$$

$$= \left(-16.3\hat{\mathbf{i}} - 8.16\hat{\mathbf{j}}\right)\mathrm{m} \approx \boxed{\left(-16.3\hat{\mathbf{i}} - 8.2\hat{\mathbf{j}}\right)\mathrm{m}}$$

41. (II) Extreme-sports enthusiasts have been known to jump off the top of El Capitan, a sheer granite cliff of height 910 m in Yosemite National Park. Assume a jumper runs horizontally off the top of El Capitan with speed 5.0 m/s and enjoys a freefall until she is 150 m above the valley floor, at which time she opens her parachute (Fig. 3–41). (*a*) How long is the jumper in freefall? Ignore air resistance. (*b*) It is important to be as far away from the cliff as possible before opening the parachute. How far from the cliff is this jumper when she opens her chute?

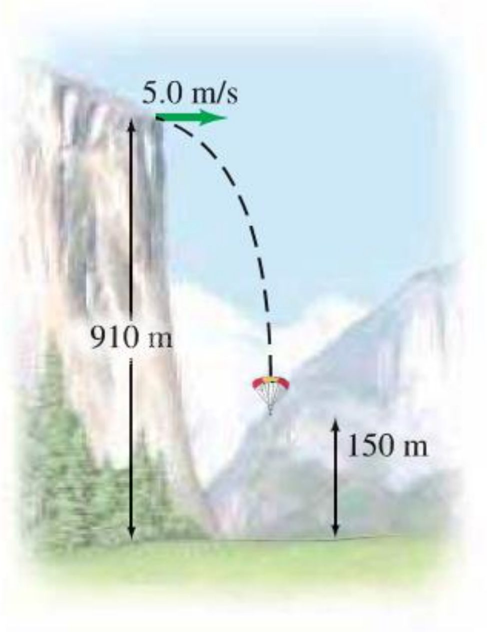


FIGURE 3–41
Problem 41.

41. (*a*) Take the ground to be the $y = 0$ level, with upward as the positive direction. Use Eq. 2-12b to solve for the time, with an initial vertical velocity of 0.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 150\,\text{m} = 910\,\text{m} + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2(150-910)}{\left(-9.80\,\text{m/s}^2\right)}} = 12.45\,\text{s} \approx \boxed{12\,\text{s}}$$

(*b*) The horizontal motion is at a constant speed, since air resistance is being ignored.

$$\Delta x = v_x t = (5.0\,\text{m/s})(12.45\,\text{s}) = 62.25\,\text{m} \approx \boxed{62\,\text{m}}$$

44. (II) (*a*) A long jumper leaves the ground at 45° above the horizontal and lands 8.0 m away. What is her "takeoff" speed v_0? (*b*) Now she is out on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5 m, vertically below. If she long jumps from the edge of the left bank at 45° with the speed calculated in (*a*), how long, or short, of the opposite bank will she land (Fig. 3–43)?

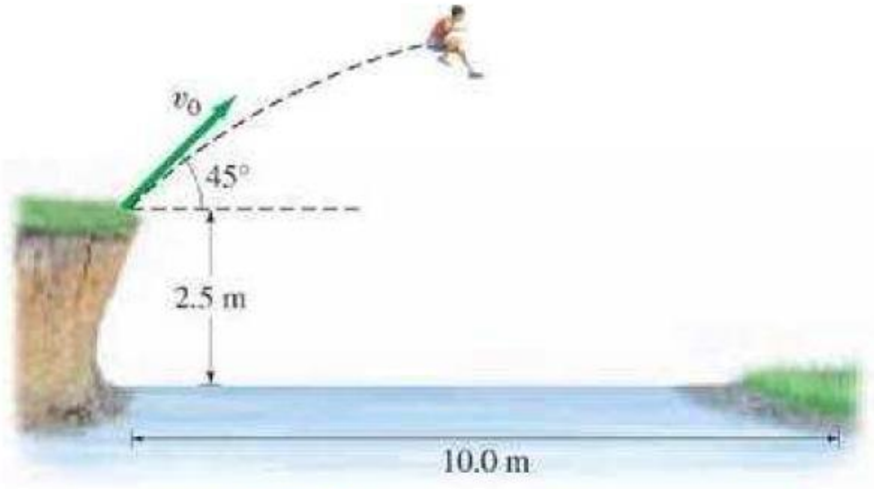


FIGURE 3–43 Problem 44.

44. (*a*) Use the "level horizontal range" formula from Example 3-10 to find her takeoff speed.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \rightarrow v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80\,\text{m/s}^2)(8.0\,\text{m})}{\sin 90°}} = 8.854\,\text{m/s} \approx \boxed{8.9\,\text{m/s}}$$

(*b*) Let the launch point be at the $y = 0$ level, and choose upward to be positive. Use Eq. 2-12b to solve for the time to fall to 2.5 meters below the starting height, and then calculate the horizontal distance traveled.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \rightarrow -2.5\,\text{m} = (8.854\,\text{m/s})\sin 45° t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2$$

$$4.9t^2 - 6.261t - 2.5\,\text{m} = 0 \rightarrow$$

$$t = \frac{6.261 \pm \sqrt{(6.261)^2 - 4(4.9)(-2.5)}}{2(4.9)} = \frac{6.261 \pm 9.391}{2(4.9)} = -0.319\,\text{s},\ 1.597\,\text{s}$$

Use the positive time to find the horizontal displacement during the jump.

$$\Delta x = v_{0x}t = v_0 \cos 45° t = (8.854\,\text{m/s})\cos 45°(1.597\,\text{s}) = 10.0\,\text{m}$$

She will land exactly on the opposite bank, neither long nor short.

46. (II) A projectile is shot from the edge of a cliff 115 m abo ground level with an initial speed of 65.0 m/s at an angle 35.0° with the horizontal, as shown in Fig. 3–44. (*a*) Dete mine the time taken by the projectile to hit point P ground level. (*b*) Determine the distance X of point from the base of the vertical cliff. At the instant ju before the projectile hits point P, find (*c*) the horizontal ar the vertical components of its velocity, (*d*) the magnitude the velocity, and (*e*) the angle made by the velocity vect with the horizontal. (*f*) Find the maximum height above tl cliff top reached by the projectile.

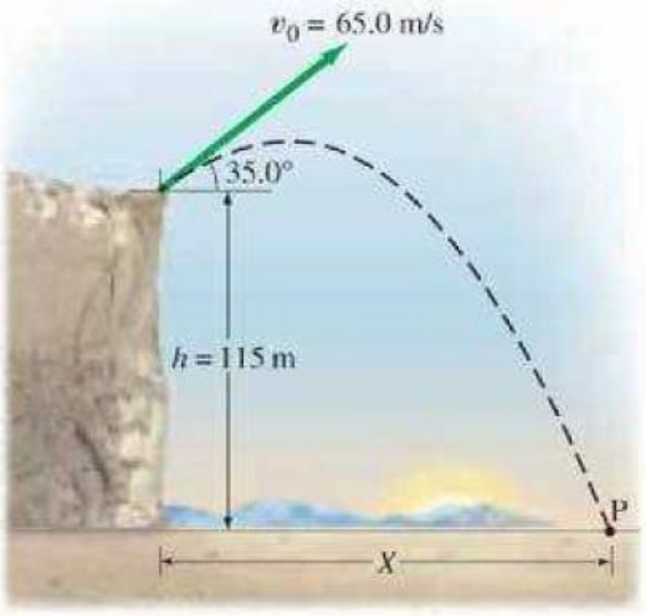


FIGURE 3–44 Problem 46.

46. Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive y direction. For the projectile, $v_0 = 65.0\,\text{m/s}$, $\theta_0 = 35.0°$, $a_y = -g$, $y_0 = 115\,\text{m}$, and $v_{y0} = v_0 \sin\theta_0$.

(*a*) The time taken to reach the ground is found from Eq. 2-12b, with a final height of 0.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow 0 = y_0 + v_0\sin\theta_0 t - \tfrac{1}{2}gt^2 \rightarrow$$

$$t = \frac{-v_0\sin\theta_0 \pm \sqrt{v_0^2\sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)} = 9.964\,\text{s}\,,\ -2.3655\,\text{s} = \boxed{9.96\,\text{s}}$$

Choose the positive time since the projectile was launched at time $t = 0$.

(*b*) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0\cos\theta_0)t = (65.0\,\text{m/s})(\cos 35.0°)(9.964\,\text{s}) = \boxed{531\,\text{m}}$$

(*c*) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0\cos\theta_0 = (65.0\,\text{m/s})\cos 35.0° = \boxed{53.2\,\text{m/s}}$. The vertical component is found from Eq. 2-12a.

$$v_y = v_{y0} + at = v_0\sin\theta_0 - gt = (65.0\,\text{m/s})\sin 35.0° - (9.80\,\text{m/s}^2)(9.964\,\text{s})$$
$$= \boxed{-60.4\,\text{m/s}}$$

(*d*) The magnitude of the velocity is found from the x and y components calculated in part (*c*) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(53.2\,\text{m/s})^2 + (-60.4\,\text{m/s})^2} = \boxed{80.5\,\text{m/s}}$$

(*e*) The direction of the velocity is $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{-60.4}{53.2} = -48.6°$, and so the object is moving $\boxed{48.6° \text{ below the horizon}}$.

(*f*) The maximum height above the cliff top reached by the projectile will occur when the y-velocity is 0, and is found from Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \rightarrow 0 = v_0^2\sin^2\theta_0 - 2gy_{max}$$

$$y_{max} = \frac{v_0^2\sin^2\theta_0}{2g} = \frac{(65.0\,\text{m/s})^2\sin^2 35.0°}{2(9.80\,\text{m/s}^2)} = \boxed{70.9\,\text{m}}$$

85. Spymaster Chris, flying a constant 208 km/h horizontally in a low-flying helicopter, wants to drop secret documents into her contact's open car which is traveling 156 km/h on a level highway 78.0 m below. At what angle (with the horizontal) should the car be in her sights when the packet is released (Fig. 3–59)?

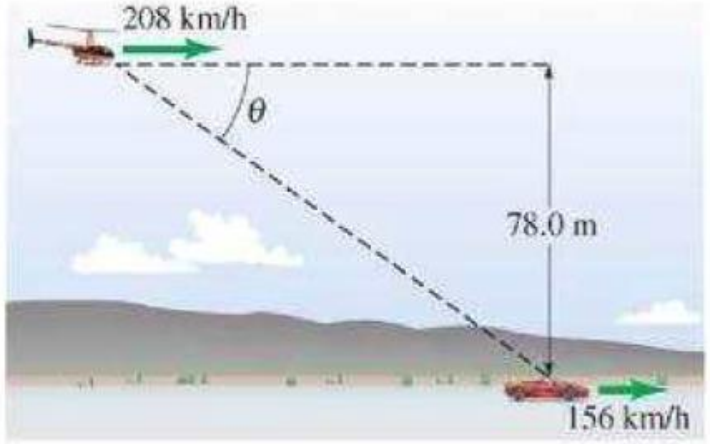


FIGURE 3–59
Problem 85.

85. Work in the frame of reference in which the car is at rest at ground level. In this reference frame, the helicopter is moving horizontally with a speed of $208\,\text{km/h} - 156\,\text{km/h} = 52\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)$ $= 14.44\,\text{m/s}$. For the vertical motion, choose the level of the helicopter to be the origin, and downward to be positive. Then the package's y displacement is $y = 78.0\text{ m}$, $v_{y0} = 0$, and $a_y = g$. The time for the package to fall is calculated from Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 78.0\text{ m} = \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)t^2 \quad\rightarrow\quad t = \sqrt{\frac{2(78.0\text{ m})}{9.80\,\text{m/s}^2}} = 3.99\text{ sec}$$

The horizontal distance that the package must move, relative to the "stationary" car, is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (14.44\,\text{m/s})(3.99\text{ s}) = 57.6\text{ m}$$

Thus the angle under the horizontal for the package release will be as follows.

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{78.0\text{ m}}{57.6\text{ m}}\right) = 53.6^\circ \approx \boxed{54^\circ}$$

55. (III) A person stands at the base of a hill that is a straight incline making an angle ϕ with the horizontal (Fig. 3–48). For a given initial speed v_0, at what angle θ (to the horizontal) should objects be thrown so that the distance d they land up the hill is as large as possible?

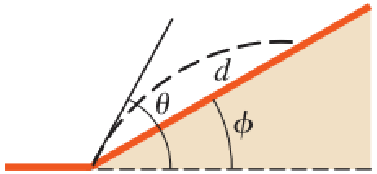


FIGURE 3–48 Problem 55. Given ϕ and v_0, determine θ to make d maximum.

55. Choose the origin to be at the bottom of the hill, just where the incline starts. The equation of the line describing the hill is $y_2 = x\tan\phi$. The equations of the motion of the object are $y_1 = v_{0y}t + \frac{1}{2}a_y t^2$ and $x = v_{0x}t$, with $v_{0x} = v_0\cos\theta$ and $v_{0y} = v_0\sin\theta$. Solve the horizontal equation for the time of flight, and insert that into the vertical projectile motion equation.

$$t = \frac{x}{v_{0x}} = \frac{x}{v_0\cos\theta} \rightarrow y_1 = v_0\sin\theta\frac{x}{v_0\cos\theta} - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2 = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$$

Equate the y-expressions for the line and the parabola to find the location where the two x-coordinates intersect.

$$x\tan\phi = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta} \rightarrow \tan\theta - \tan\phi = \frac{gx}{2v_0^2\cos^2\theta} \rightarrow$$

$$x = \frac{(\tan\theta - \tan\phi)2v_0^2\cos^2\theta}{g}$$

This intersection x-coordinate is related to the desired quantity d by $x = d\cos\phi$.

$$d\cos\phi = (\tan\theta - \tan\phi)\frac{2v_0^2\cos^2\theta}{g} \quad \rightarrow \quad d = \frac{2v_0^2}{g\cos\phi}\left(\sin\theta\cos\theta - \tan\phi\cos^2\theta\right)$$

To maximize the distance, set the derivative of d with respect to θ equal to 0, and solve for θ.

$$\frac{d(d)}{d\theta} = \frac{2v_0^2}{g\cos\phi}\frac{d}{d\theta}\left(\sin\theta\cos\theta - \tan\phi\cos^2\theta\right)$$

$$= \frac{2v_0^2}{g\cos\phi}\left[\sin\theta(-\sin\theta) + \cos\theta(\cos\theta) - \tan\phi(2)\cos\theta(-\sin\theta)\right]$$

$$= \frac{2v_0^2}{g\cos\phi}\left[-\sin^2\theta + \cos^2\theta + 2\tan\phi\cos\theta\sin\theta\right] = \frac{2v_0^2}{g\cos\phi}\left[\cos 2\theta + \sin 2\theta\tan\phi\right] = 0$$

$$\cos 2\theta + \sin 2\theta\tan\phi = 0 \quad \rightarrow \quad \theta = \tfrac{1}{2}\tan^{-1}\left(-\frac{1}{\tan\phi}\right)$$

This expression can be confusing, because it would seem that a negative sign enters the solution. In order to get appropriate values, 180° or π radians must be added to the angle resulting from the inverse tangent operation, to have a positive angle. Thus a more appropriate expression would be the following:

$\theta = \tfrac{1}{2}\left[\pi + \tan^{-1}\left(-\frac{1}{\tan\phi}\right)\right]$. This can be shown to be equivalent to $\boxed{\theta = \frac{\phi}{2} + \frac{\pi}{4}}$, because

$$\tan^{-1}\left(-\frac{1}{\tan\phi}\right) = \tan^{-1}(-\cot\phi) = \cot^{-1}\cot\phi - \frac{\pi}{2} = \phi - \frac{\pi}{2}.$$