*15. A thin uniform rod of mass m is suspended from one end and oscillates with a frequency f. If a small sphere of mass $2m$ is attached to the other end, does the frequency increase or decrease? Explain.

15. The frequency will decrease. For a physical pendulum, the period is proportional to the square root of the moment of inertia divided by the mass. When the small sphere is added to the end of the rod, both the moment of inertia and the mass of the pendulum increase. However, the increase in the moment of inertia will be greater because the added mass is located far from the axis of rotation. Therefore, the period will increase and the frequency will decrease.

2. (I) An elastic cord is 65 cm long when a weight of 75 N hangs from it but is 85 cm long when a weight of 180 N hangs from it. What is the "spring" constant k of this elastic cord?

2. The spring constant is the ratio of external applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{180\text{ N} - 75\text{ N}}{0.85\text{ m} - 0.65\text{ m}} = \frac{105\text{ N}}{0.20\text{ m}} = 525\text{ N/m} \approx \boxed{530\text{ N/m}}$$

9. (II) A small fly of mass 0.25 g is caught in a spider's web. The web oscillates predominately with a frequency of 4.0 Hz. (*a*) What is the value of the effective spring stiffness constant k for the web? (*b*) At what frequency would you expect the web to oscillate if an insect of mass 0.50 g were trapped?

9. The relationship between frequency, mass, and spring constant is Eq. 14-7a, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(*a*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow k = 4\pi^2 f^2 m = 4\pi^2 (4.0\text{ Hz})^2 (2.5\times10^{-4}\text{ kg}) = 0.1579\text{ N/m} \approx \boxed{0.16\text{ N/m}}$$

(*b*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.1579\text{ N/m}}{5.0\times10^{-4}\text{ kg}}} = \boxed{2.8\text{ Hz}}$$

13. (II) Figure 14–29 shows two examples of SHM, labeled A and B. For each, what is (*a*) the amplitude, (*b*) the frequency, and (*c*) the period? (*d*) Write the equations for both A and B in the form of a sine or cosine.

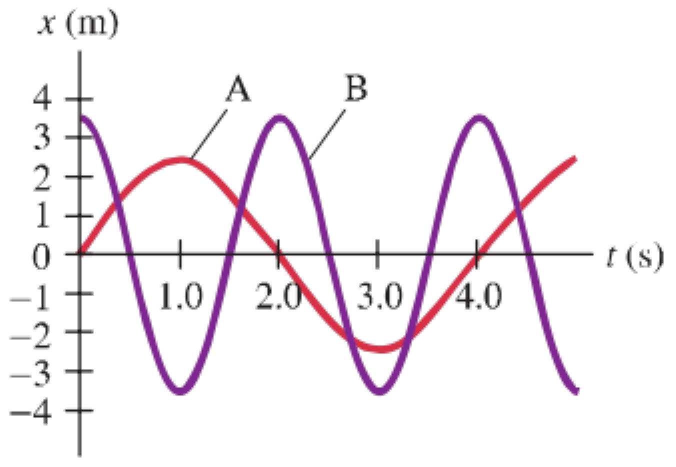

FIGURE 14–29 Problem 13.

13. (*a*) For A, the amplitude is $A_A = \boxed{2.5\,\text{m}}$. For B, the amplitude is $A_B = \boxed{3.5\,\text{m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_A = \boxed{0.25\,\text{Hz}}$. For B, the frequency is 1 cycle every 2.0 seconds, so $f_B = \boxed{0.50\,\text{Hz}}$.

(*c*) For C, the period is $T_A = \boxed{4.0\,\text{s}}$. For B, the period is $T_B = \boxed{2.0\,\text{s}}$

(*d*) Object A has a displacement of 0 when $t = 0$, so it is a sine function.

$$x_A = A_A \sin(2\pi f_A t) \quad \rightarrow \quad \boxed{x_A = (2.5\,\text{m})\sin\left(\tfrac{1}{2}\pi t\right)}$$

Object B has a maximum displacement when $t = 0$, so it is a cosine function.

$$x_B = A_B \cos(2\pi f_B t) \quad \rightarrow \quad \boxed{x_B = (3.5\,\text{m})\cos(\pi t)}$$

16. (II) The graph of displacement vs. time for a small mass m at the end of a spring is shown in Fig. 14–30. At $t = 0$, $x = 0.43$ cm. (*a*) If $m = 9.5$ g, find the spring constant, k. (*b*) Write the equation for displacement x as a function of time.

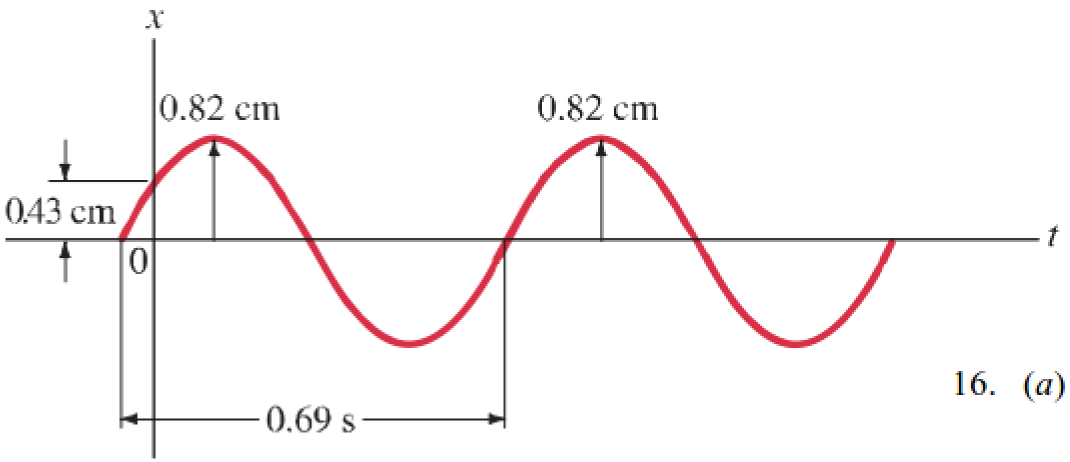

FIGURE 14–30 Problem 16.

16. (*a*) From the graph, the period is 0.69 s. The period and the mass can be used to find the spring constant.

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow k = 4\pi^2\frac{m}{T^2} = 4\pi^2\frac{0.0095\,\text{kg}}{(0.69\,\text{s})^2} = 0.7877\,\text{N/m} \approx \boxed{0.79\,\text{N/m}}$$

(*b*) From the graph, the amplitude is 0.82 cm. The phase constant can be found from the initial conditions.

$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right) = (0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t + \phi\right)$$

$$x(0) = (0.82\,\text{cm})\cos\phi = 0.43\,\text{cm} \rightarrow \phi = \cos^{-1}\frac{0.43}{0.82} = \pm 1.02\,\text{rad}$$

Because the graph is shifted to the RIGHT from the 0-phase cosine, the phase constant must be subtracted.

$$x = \boxed{(0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t - 1.0\right)} \text{ or } (0.82\,\text{cm})\cos(9.1t - 1.0)$$

25. (III) A mass m is connected to two springs, with spring constants k_1 and k_2, in two different ways as shown in Fig. 14–32a and b. Show that the period for the configuration shown in part (a) is given by

$$T = 2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}$$

and for that in part (b) is given by

$$T = 2\pi\sqrt{\frac{m}{k_1+k_2}}.$$

Ignore friction.

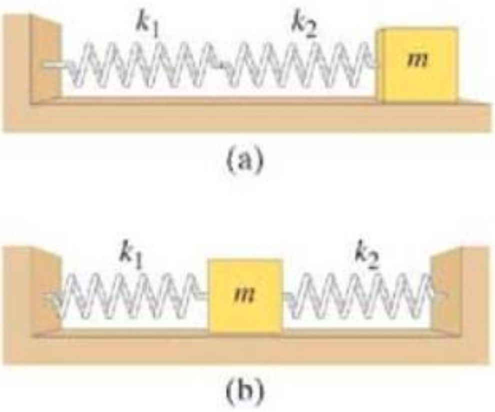

FIGURE 14–32 Problem 25.

25. (a) If the block is displaced a distance x to the right in Figure 14-32a, then the length of spring # 1 will be increased by a distance x_1 and the length of spring # 2 will be increased by a distance x_2, where $x = x_1 + x_2$. The force on the block can be written $F = -k_{\text{eff}}x$. Because the springs are massless, they act similar to a rope under tension, and the same force F is exerted by each spring. Thus $F = -k_{\text{eff}}x = -k_1x_1 = -k_2x_2$.

$$x = x_1 + x_2 = -\frac{F}{k_1} - \frac{F}{k_2} = -F\left(\frac{1}{k_1}+\frac{1}{k_2}\right) = -\frac{F}{k_{\text{eff}}} \rightarrow \frac{1}{k_{\text{eff}}} = \frac{1}{k_1}+\frac{1}{k_2}$$

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}} = \boxed{2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}}$$

(b) The block will be in equilibrium when it is stationary, and so the net force at that location is zero. Then, if the block is displaced a distance x to the right in the diagram, then spring # 1 will exert an additional force of $F_1 = -k_1x$, in the opposite direction to x. Likewise, spring # 2 will exert an additional force $F_2 = -k_2x$, in the same direction as F_1. Thus the net force on the displaced block is $F = F_1 + F_2 = -k_1x - k_2x = -(k_1 + k_2)x$. The effective spring constant is thus $k = k_1 + k_2$, and the period is given by $T = 2\pi\sqrt{\frac{m}{k}} = \boxed{2\pi\sqrt{\frac{m}{k_1+k_2}}}$.

32. (II) A 0.0125-kg bullet strikes a 0.240-kg block attached to a fixed horizontal spring whose spring constant is 2.25×10^3 N/m and sets it into oscillation with an amplitude of 12.4 cm. What was the initial speed of the bullet if the two objects move together after impact?

32. The energy of the oscillator will be conserved after the collision.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(m+M)v_{max}^2 \rightarrow v_{max} = A\sqrt{k/(m+M)}$$

This speed is the speed that the block and bullet have immediately after the collision. Linear momentum in one dimension will have been conserved during the (assumed short time) collision, and so the initial speed of the bullet can be found.

$$p_{before} = p_{after} \rightarrow mv_o = (m+M)v_{max}$$

$$v_o = \frac{m+M}{m}A\sqrt{\frac{k}{m+M}} = \frac{0.2525\,\text{kg}}{0.0125\,\text{kg}}(0.124\,\text{m})\sqrt{\frac{2250\,\text{N/m}}{0.2525\,\text{kg}}} = \boxed{236\,\text{m/s}}$$

37. (II) Agent Arlene devised the following method of measuring the muzzle velocity of a rifle (Fig. 14–33). She fires a bullet into a 4.648-kg wooden block resting on a smooth surface, and attached to a spring of spring constant $k = 142.7$ N/m. The bullet, whose mass is 7.870 g, remains embedded in the wooden block. She measures the maximum distance that the block compresses the spring to be 9.460 cm. What is the speed v of the bullet?

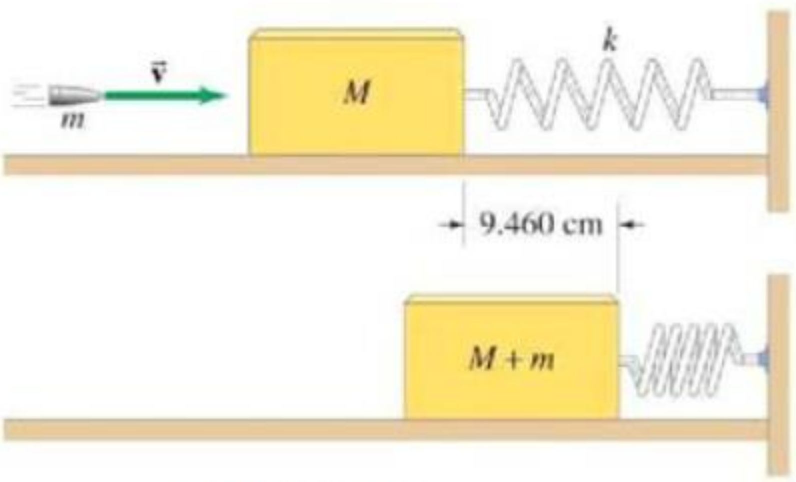

FIGURE 14–33 Problem 37.

37. We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then, the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$p_{before} = p_{after} \rightarrow mv_0 = (m+M)v_{max} \rightarrow v_{max} = \frac{m}{m+M}v_0$$

$$\tfrac{1}{2}(m+M)v_{max}^2 = \tfrac{1}{2}kA^2 \rightarrow \tfrac{1}{2}(m+M)\left(\frac{m}{m+M}v_0\right)^2 = \tfrac{1}{2}kA^2 \rightarrow$$

$$v_0 = \frac{A}{m}\sqrt{k(m+M)} = \frac{(9.460\times10^{-2}\,\text{m})}{(7.870\times10^{-3}\,\text{kg})}\sqrt{(142.7\,\text{N/m})(7.870\times10^{-3}\,\text{kg}+4.648\,\text{kg})}$$

$$= \boxed{309.8\,\text{m/s}}$$

76. An oxygen atom at a particular site within a DNA molecule can be made to execute simple harmonic motion when illuminated by infrared light. The oxygen atom is bound with a spring-like chemical bond to a phosphorus atom, which is rigidly attached to the DNA backbone. The oscillation of the oxygen atom occurs with frequency $f = 3.7 \times 10^{13}$ Hz. If the oxygen atom at this site is chemically replaced with a sulfur atom, the spring constant of the bond is unchanged (sulfur is just below oxygen in the Periodic Table). Predict the frequency for a DNA molecule after the sulfur substitution.

76. The spring constant does not change, but the mass does, and so the frequency will change. Use Eq. 14-7a to relate the spring constant, the mass, and the frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{k}{4\pi^2} = f^2 m = \text{constant} \rightarrow f_O^2 m_O = f_S^2 m_S \rightarrow$$

$$f_S = f_O\sqrt{\frac{m_O}{m_S}} = \left(3.7\times10^{13}\,\text{Hz}\right)\sqrt{\frac{16.0}{32.0}} = \boxed{2.6\times10^{13}\,\text{Hz}}$$

86. Carbon dioxide is a linear molecule. The carbon–oxygen bonds in this molecule act very much like springs. Figure 14–43 shows one possible way the oxygen atoms in this molecule can oscillate: the oxygen atoms oscillate symmetrically in and out, while the central carbon atom remains at rest. Hence each oxygen atom acts like a simple harmonic oscillator with a mass equal to the mass of an oxygen atom. It is observed that this oscillation occurs with a frequency of $f = 2.83 \times 10^{13}$ Hz. What is the spring constant of the C—O bond?

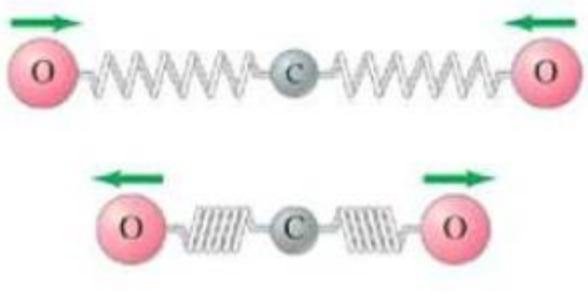

FIGURE 14–43 Problem 86, the CO_2 molecule.

86. The effective spring constant is determined by the frequency of vibration and the mass of the oscillator. Use Eq. 14-7a.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow$$

$$k = 4\pi^2 f^2 m = 4\pi^2\left(2.83\times10^{13}\,\text{Hz}\right)\left(16.00\,\text{u}\right)\left(\frac{1.66\times10^{-27}\,\text{kg}}{1\,\text{u}}\right) = \boxed{840\,\text{N/m}}\ \text{(3 sig. fig.)}$$