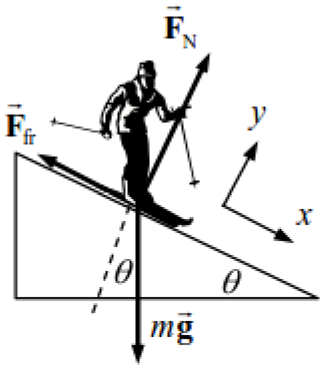
9. (II) A skier moves down a 27° slope at constant speed. What can you say about the coefficient of friction, μ_k? Assume the speed is low enough that air resistance can be ignored.

9. Since the skier is moving at a constant speed, the net force on the skier must be 0. See the free-body diagram, and write Newton's second law for both the x and y directions.

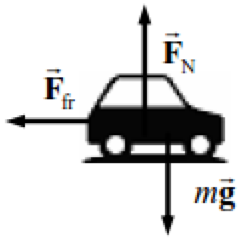
$$mg\sin\theta = F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg\cos\theta \ \rightarrow$$

$$\mu_s = \tan\theta = \tan 27° = \boxed{0.51}$$



14. (II) Police investigators, examining the scene of an accident involving two cars, measure 72-m-long skid marks of one of the cars, which nearly came to a stop before colliding. The coefficient of kinetic friction between rubber and the pavement is about 0.80. Estimate the initial speed of that car assuming a level road.

14. Assume that kinetic friction is the net force causing the deceleration. See the free-body diagram for the car, assuming that the right is the positive direction, and the direction of motion of the skidding car. There is no acceleration in the vertical direction, and so $F_{\text{N}} = mg$. Applying Newton's second law to the x direction gives the following.



$$\sum F = -F_f = ma \ \rightarrow \ -\mu_k F_N = -\mu_k mg = ma \ \rightarrow \ a = -\mu_k g$$

Use Eq. 2-12c to determine the initial speed of the car, with the final speed of the car being zero.

$$v^2 - v_0^2 = 2a(x - x_0) \ \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{0 - 2(-\mu_k g)(x - x_0)} = \sqrt{2(0.80)(9.80\,\text{m/s}^2)(72\ \text{m})} = \boxed{34\,\text{m/s}}$$

20. (II) Two blocks made of different materials connected together by a thin cord, slide down a plane ramp inclined at an angle θ to the horizontal as shown in Fig. 5–34 (block B is above block A). The masses of the blocks are m_A and m_B, and the coefficients of friction are μ_A and μ_B. If $m_A = m_B = 5.0$ kg, and $\mu_A = 0.20$ and $\mu_B = 0.30$, determine (*a*) the acceleration of the blocks and (*b*) the tension in the cord, for an angle $\theta = 32°$.

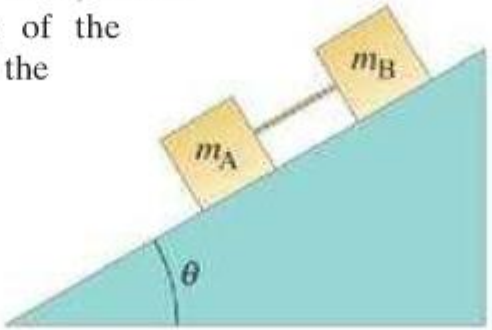
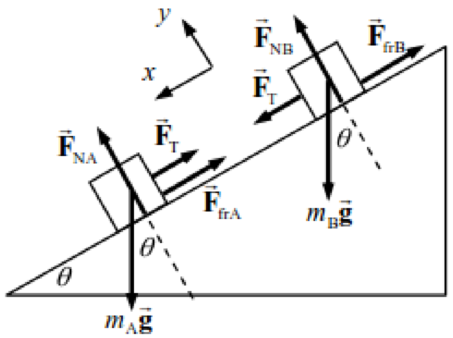


FIGURE 5–34
Problems 20 and 21.

20. Since the upper block has a higher coefficient of friction, that block will "drag behind" the lower block. Thus there will be tension in the cord, and the blocks will have the same acceleration. From the free-body diagrams for each block, we write Newton's second law for both the *x* and *y* directions for each block, and then combine those equations to find the acceleration and tension.



(*a*) Block A:

$$\sum F_{yA} = F_{NA} - m_A g\cos\theta = 0 \rightarrow F_{NA} = m_A g\cos\theta$$

$$\sum F_{xA} = m_A g\sin\theta - F_{frA} - F_T = m_A a$$

$$m_A a = m_A g\sin\theta - \mu_A F_{NA} - F_T = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T$$

Block B:

$$\sum F_{yB} = F_{NB} - m_B g\cos\theta = 0 \rightarrow F_{NB} = m_B g\cos\theta$$

$$\sum F_{xB} = m_A g\sin\theta - F_{frA} + F_T = m_B a$$

$$m_B a = m_B g\sin\theta - \mu_B F_{NB} + F_T = m_B g\sin\theta - \mu_B m_B g\cos\theta + F_T$$

Add the final equations together from both analyses and solve for the acceleration.

$$m_A a = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T \ ; \ m_B a = m_B g\sin\theta - \mu_B m_B g\cos\theta + F_T$$

$$m_A a + m_B a = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T + m_B g\sin\theta - \mu_B m_B g\cos\theta + F_T \rightarrow$$

$$a = g\left[\frac{m_A(\sin\theta - \mu_A\cos\theta) + m_B(\sin\theta - \mu_B\cos\theta)}{(m_A + m_B)}\right]$$

$$= (9.80\,\text{m/s}^2)\left[\frac{(5.0\,\text{kg})(\sin 32° - 0.20\cos 32°) + (5.0\,\text{kg})(\sin 32° - 0.30\cos 32°)}{(10.0\,\text{kg})}\right]$$

$$= 3.1155\,\text{m/s}^2 \approx \boxed{3.1\,\text{m/s}^2}$$

(*b*) Solve one of the equations for the tension force.

$$m_A a = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T \rightarrow$$

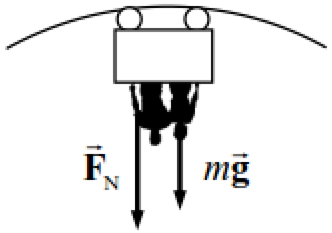
$$F_T = m_A(g\sin\theta - \mu_A g\cos\theta - a)$$

$$= (5.0\,\text{kg})\left[(9.80\,\text{m/s}^2)(\sin 32° - 0.20\cos 32°) - 3.1155\,\text{m/s}^2\right] = \boxed{2.1\,\text{N}}$$

40. (II) At what minimum speed must a roller coaster be traveling when upside down at the top of a circle (Fig. 5–42) so that the passengers do not fall out? Assume a radius of curvature of 7.6 m.

FIGURE 5–42 Problem 40.

40. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's second law for the passengers.



$$\sum F = F_N + mg = ma = mv^2/r \quad \rightarrow \quad F_N = m\left(v^2/r - g\right)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car – they are in free fall. The limiting condition is as follows.

$$v_{min}^2/r - g = 0 \quad \rightarrow \quad v_{min} = \sqrt{rg} = \sqrt{\left(9.80\,\text{m/s}^2\right)\left(7.6\,\text{m}\right)} = \boxed{8.6\,\text{m/s}}$$

54. (II) Two blocks, with masses m_A and m_B, are connected to each other and to a central post by cords as shown in Fig. 5–46. They rotate about the post at frequency f (revolutions per second) on a frictionless horizontal surface at distances r_A and r_B from the post. Derive an algebraic expression for the tension in each segment of the cord (assumed massless).

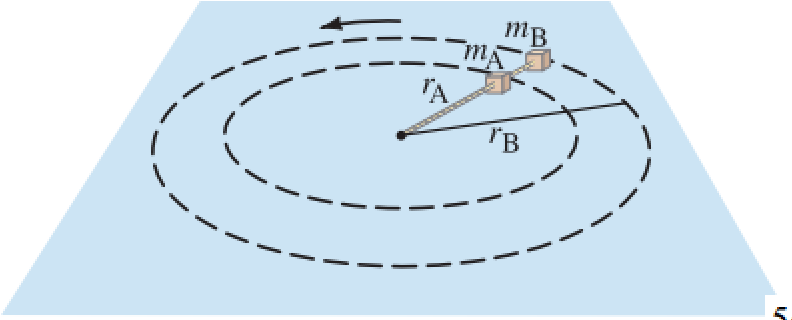
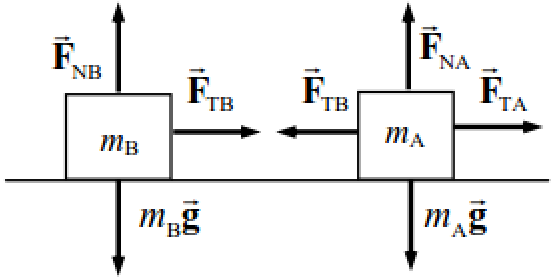


FIGURE 5–46 Problem 54.

54. If the masses are in line and both have the same frequency of rotation, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's third law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's second law for the horizontal direction for both masses, noting that they are in uniform circular motion.



$$\sum F_{RA} = F_{TA} - F_{TB} = m_A a_A = m_A v_A^2/r_A \qquad \sum F_{RB} = F_{TB} = m_B a_B = m_B v_B^2/r_B$$

The speeds can be expressed in terms of the frequency as follows: $v = \left(f\,\frac{\text{rev}}{\text{sec}}\right)\left(\frac{2\pi r}{1\text{ rev}}\right) = 2\pi r f$.

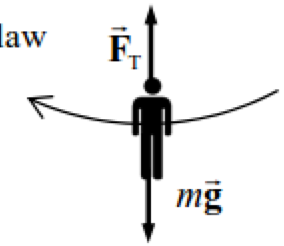
$$F_{TB} = m_B v_B^2/r_B = m_B\left(2\pi r_B f\right)^2/r_B = \boxed{4\pi^2 m_B r_B f^2}$$

$$F_{TA} = F_{TB} + m_A v_A^2/r_A = 4\pi m_B r_B f^2 + m_A\left(2\pi r_A f\right)^2/r_A = \boxed{4\pi^2 f^2\left(m_A r_A + m_B r_B\right)}$$

55. (II) Tarzan plans to cross a gorge by swinging in an arc from a hanging vine (Fig. 5–47). If his arms are capable of exerting a force of 1350 N on the rope, what is the maximum speed he can tolerate at the lowest point of his swing? His mass is 78 kg and the vine is 5.2 m long.

FIGURE 5–47 Problem 55.

55. A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's second law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal, and points upward when he is at the bottom.



$$\sum F = F_T - mg = ma = mv^2/r \rightarrow v = \sqrt{\frac{(F_T - mg)r}{m}}$$

The maximum speed will be obtained with the maximum tension.

$$v_{max} = \sqrt{\frac{(\vec{F}_{T\,max} - mg)r}{m}} = \sqrt{\frac{(1350\text{ N} - (78\text{ kg})(9.80\text{ m/s}^2))5.2\text{ m}}{78\text{ kg}}} = \boxed{6.2\text{ m/s}}$$

57. (III) The position of a particle moving in the xy plane is given by $\vec{\mathbf{r}} = 2.0\cos(3.0\text{ rad/s}\ t)\hat{\mathbf{i}} + 2.0\sin(3.0\text{ rad/s}\ t)\hat{\mathbf{j}}$, where r is in meters and t is in seconds. (*a*) Show that this represents circular motion of radius 2.0 m centered at the origin. (*b*) Determine the velocity and acceleration vectors as functions of time. (*c*) Determine the speed and magnitude of the acceleration. (*d*) Show that $a = v^2/r$. (*e*) Show that the acceleration vector always points toward the center of the circle.

57. (*a*) We are given that $x = (2.0\text{ m})\cos(3.0\text{ rad/s}\ t)$ and $y = (2.0\text{ m})\sin(3.0\text{ rad/s}\ t)$. Square both components and add them together.

$$x^2 + y^2 = \left[(2.0\text{ m})\cos(3.0\text{ rad/s}\ t)\right]^2 + \left[(2.0\text{ m})\sin(3.0\text{ rad/s}\ t)\right]^2$$

$$= (2.0\text{ m})^2\left[\cos^2(3.0\text{ rad/s}\ t) + \sin^2(3.0\text{ rad/s}\ t)\right] = (2.0\text{ m})^2$$

This is the equation of a circle, $x^2 + y^2 = r^2$, with a radius of 2.0 m.

(*b*) $$\boxed{\vec{\mathbf{v}} = (-6.0\text{ m/s})\sin(3.0\text{ rad/s}\ t)\hat{\mathbf{i}} + (6.0\text{ m/s})\cos(3.0\text{ rad/s}\ t)\hat{\mathbf{j}}}$$

$$\boxed{\vec{\mathbf{a}} = (-18\text{ m/s}^2)\cos(3.0\text{ rad/s}\ t)\hat{\mathbf{i}} + (-18\text{ m/s}^2)\sin(3.0\text{ rad/s}\ t)\hat{\mathbf{j}}}$$

(*c*) $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left[(-6.0\text{ m/s})\sin(3.0\text{ rad/s}\ t)\right]^2 + \left[(6.0\text{ m/s})\cos(3.0\text{ rad/s}\ t)\right]^2} = \boxed{6.0\text{ m/s}}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left[(-18\text{ m/s}^2)\cos(3.0\text{ rad/s}\ t)\right]^2 + \left[(-18\text{ m/s}^2)\sin(3.0\text{ rad/s}\ t)\right]^2} = \boxed{18\text{ m/s}^2}$$

(*d*) $$\frac{v^2}{r} = \frac{(6.0\text{ m/s})^2}{2.0\text{ m}} = 18\text{ m/s}^2 = a$$

(*e*) $$\vec{\mathbf{a}} = (-18\text{ m/s}^2)\cos(3.0\text{ rad/s}\ t)\hat{\mathbf{i}} + (-18\text{ m/s}^2)\sin(3.0\text{ rad/s}\ t)\hat{\mathbf{j}}$$

$$= (-9.0/\text{s}^2)\left[2.0\text{ m}\cos(3.0\text{ rad/s}\ t)\hat{\mathbf{i}} + 2.0\text{ m}\sin(3.0\text{ rad/s}\ t)\hat{\mathbf{j}}\right] = (9.0/\text{s}^2)(-\vec{\mathbf{r}})$$

We see that the acceleration vector is directed oppositely of the position vector. Since the position vector points outward from the center of the circle, the acceleration vector points toward the center of the circle.

80. A flat puck (mass M) is revolved in a circle on a frictionless air hockey table top, and is held in this orbit by a light cord which is connected to a dangling mass (mass m) through a central hole as shown in Fig. 5–48. Show that the speed of the puck is given by $v = \sqrt{mgR/M}$.

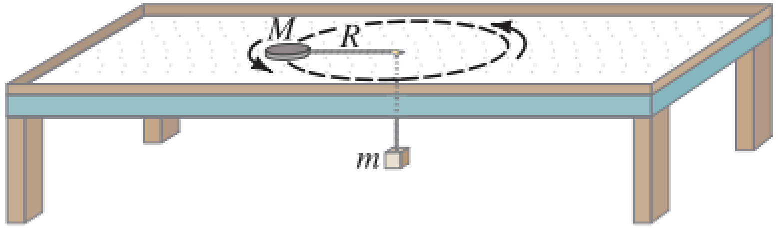


FIGURE 5–48 Problem 80.

80. Since mass m is dangling, the tension in the cord must be equal to the weight of mass m, and so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass M, and so $F_T = F_R = Ma_R = Mv^2/r$. Equate the expressions for tension and solve for the velocity.

$$Mv^2/r = mg \quad \rightarrow \quad v = \boxed{\sqrt{mgR/M}}$$

83. A device for training astronauts and jet fighter pilots is designed to rotate the trainee in a horizontal circle of radius 11.0 m. If the force felt by the trainee is 7.45 times her own weight, how fast is she rotating? Express your answer in both m/s and rev/s.

83. The force is a centripetal force, and is of magnitude $7.45mg$. Use Eq. 5-3 for centripetal force.

$$F = m\frac{v^2}{r} = 7.45mg \quad \rightarrow \quad v = \sqrt{7.45rg} = \sqrt{7.45(11.0\,\text{m})(9.80\,\text{m/s}^2)} = 28.34\,\text{m/s} \approx \boxed{28.3\,\text{m/s}}$$

$$(28.34\,\text{m/s}) \times \frac{1\,\text{rev}}{2\pi(11.0\,\text{m})} = \boxed{0.410\,\text{rev/s}}$$

87. A small mass m is set on the surface of a sphere, Fig. 5–51. If the coefficient of static friction is $\mu_s = 0.70$, at what angle ϕ would the mass start sliding?

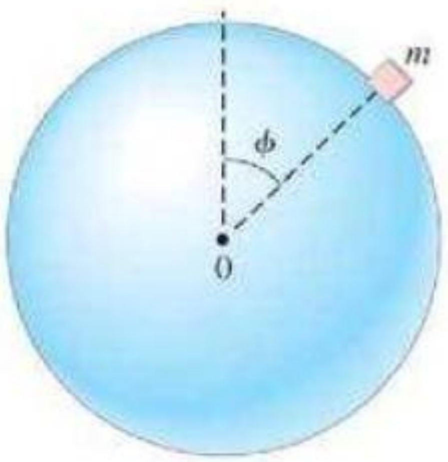
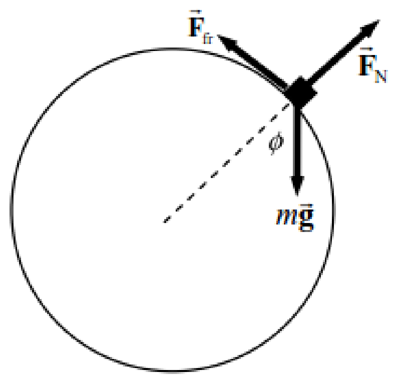


FIGURE 5–51 Problem 87.

87. The mass would start sliding when the static frictional force was not large enough to counteract the component of gravity that will be pulling the mass along the curved surface. See the free-body diagram, and assume that the static frictional force is a maximum. We also assume the block has no speed, so the radial force must be 0.



$$\sum F_{\text{radial}} = F_N - mg\cos\phi \rightarrow F_N = mg\cos\phi$$

$$\sum F_{\text{tangential}} = mg\sin\phi - F_{fr} \rightarrow F_{fr} = mg\sin\phi$$

$$F_{fr} = \mu_s F_N = \mu_s mg\cos\phi = mg\sin\phi \rightarrow \mu_s = \tan\phi \rightarrow$$

$$\phi = \tan^{-1}\mu_s = \tan^{-1}0.70 = \boxed{35^\circ}$$

93. A small bead of mass m is constrained to slide without friction inside a circular vertical hoop of radius r which rotates about a vertical axis (Fig. 5–54) at a frequency f. (*a*) Determine the angle θ where the bead will be in equilibrium—that is, where it will have no tendency to move up or down along the hoop. (*b*) If $f = 2.00$ rev/s and $r = 22.0$ cm, what is θ? (*c*) Can the bead ride as high as the center of the circle ($\theta = 90°$)? Explain.

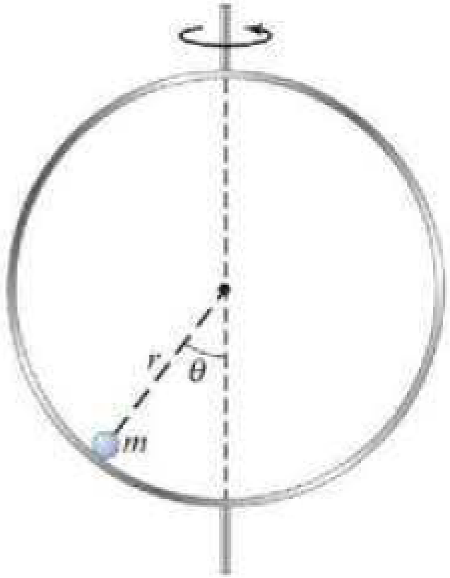
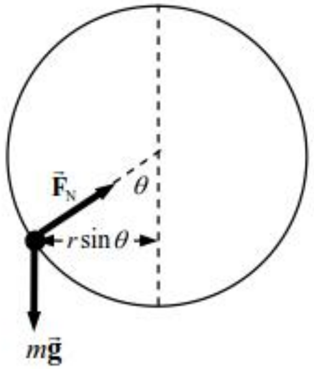


FIGURE 5–54 Problem 93.

93. (*a*) Because there is no friction between the bead and the hoop, the hoop can only exert a normal force on the bead. See the free-body diagram for the bead at the instant shown in the textbook figure. Note that the bead moves in a horizontal circle, parallel to the floor. Thus the centripetal force is horizontal, and the net vertical force must be 0. Write Newton's second law for both the horizontal and vertical directions, and use those equations to determine the angle θ. We also use the fact that the speed and the frequency are related to each other, by $v = 2\pi fr\sin\theta$.



$$\sum F_{\text{vertical}} = F_N\cos\theta - mg = 0 \rightarrow F_N = \frac{mg}{\cos\theta}$$

$$\sum F_{\text{radial}} = F_N\sin\theta = m\frac{v^2}{r\sin\theta} = m\frac{4\pi^2 f^2 r^2\sin^2\theta}{r\sin\theta}$$

$$F_N \sin\theta = \frac{mg}{\cos\theta}\sin\theta = m\frac{4\pi^2 f^2 r^2 \sin^2\theta}{r\sin\theta} \quad \rightarrow \quad \theta = \boxed{\cos^{-1}\frac{g}{4\pi^2 f^2 r}}$$

(*b*) $\theta = \cos^{-1}\frac{g}{4\pi^2 f^2 r} = \cos^{-1}\frac{9.80\,\text{m/s}^2}{4\pi^2(2.00\,\text{Hz})^2(0.220\,\text{m})} = \boxed{73.6^\circ}$

(*c*) $\boxed{\text{No}}$, the bead cannot ride as high as the center of the circle. If the bead were located there, the normal force of the wire on the bead would point horizontally. There would be no force to counteract the bead's weight, and so it would have to slip back down below the horizontal to balance the force of gravity. From a mathematical standpoint, the expression $\frac{g}{4\pi^2 f^2 r}$ would have to be equal to 0 and that could only happen if the frequency or the radius were infinitely large.

100. A ball of mass $m = 1.0\ \text{kg}$ at the end of a thin cord of length $r = 0.80\ \text{m}$ revolves in a vertical circle about point O, as shown in Fig. 5–56. During the time we observe it, the only forces acting on the ball are gravity and the tension in the cord. The motion is circular but not uniform because of the force of gravity. The ball increases in speed as it descends and decelerates as it rises on the other side of the circle. At the moment the cord makes an angle $\theta = 30°$ below the horizontal, the ball's speed is 6.0 m/s. At this point, determine the tangential acceleration, the radial acceleration, and the tension in the cord, F_T. Take θ increasing downward as shown.

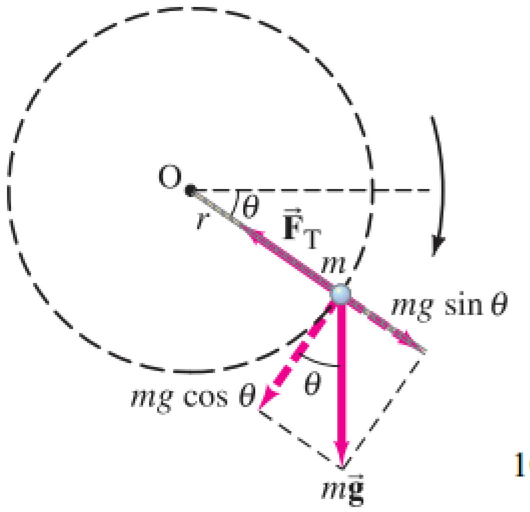


FIGURE 5–56
Problem 100.

100. The radial acceleration is $a_R = \dfrac{v^2}{r}$, and so $a_R = \dfrac{v^2}{r} = \dfrac{(6.0\,\text{m/s})^2}{0.80\,\text{m}} = \boxed{45\,\text{m/s}^2}$.

The tension force has no tangential component, and so the tangential force is seen from the diagram to be $F_{\text{tang}} = mg\cos\theta$.

$$F_{\text{tang}} = mg\cos\theta = ma_{\text{tang}} \quad\rightarrow\quad a_{\text{tang}} = g\cos\theta = (9.80\,\text{m/s}^2)\cos 30° = \boxed{8.5\,\text{m/s}^2}$$

The tension force can be found from the net radial force.

$$F_R = F_T - mg\sin\theta = m\frac{v^2}{r} \quad\rightarrow$$

$$F_T = m\left(g\sin\theta + \frac{v^2}{r}\right) = (1.0\,\text{kg})\left((9.80\,\text{m/s}^2)\sin 30° + 45\,\text{m/s}^2\right) = \boxed{50\,\text{N}}$$

Note that the answer has 2 significant figures.