4. Are there any good reasons for calling the term $\mu_0 \epsilon_0 \, d\Phi_E/dt$ in Eq. 31–1 a "current"? Explain.

4. One possible reason the term $\varepsilon_0 \frac{d\Phi_E}{dt}$ can be called a "current" is because it has units of amperes.

6. Is sound an electromagnetic wave? If not, what kind of wave is it?

7. Can EM waves travel through a perfect vacuum? Can sound waves?

6. No. Sound is a longitudinal mechanical wave. It requires the presence of a medium; electromagnetic waves do not require a medium.

7. EM waves are self-propagating and can travel through a perfect vacuum. Sound waves are mechanical waves which require a medium, and therefore cannot travel through a perfect vacuum.

16. If a radio transmitter has a vertical antenna, should a receiver's antenna (rod type) be vertical or horizontal to obtain best reception?

16. The receiver's antenna should also be vertical for the best reception.

2. (I) Calculate the displacement current I_D between the square plates, 5.8 cm on a side, of a capacitor if the electric field is changing at a rate of $2.0 \times 10^6\ \mathrm{V/m \cdot s}$.

2. The displacement current is shown in section 31-1 to be $I_D = \varepsilon_0 A \dfrac{dE}{dt}$.

$$I_D = \varepsilon_0 A \frac{dE}{dt} = \left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.058\ \mathrm{m})^2\left(2.0\times10^6\ \frac{\mathrm{V}}{\mathrm{m\bullet s}}\right) = \boxed{6.0\times10^{-8}\ \mathrm{A}}$$

6. (II) Suppose an air-gap capacitor has circular plates of radius $R = 2.5$ cm and separation $d = 1.6$ mm. A 76.0-Hz emf, $\mathscr{E} = \mathscr{E}_0 \cos \omega t$, is applied to the capacitor. The maximum displacement current is 35 μA. Determine (*a*) the maximum conduction current I, (*b*) the value of $\mathscr{E}_0$, (*c*) the maximum value of $d\Phi_E/dt$ between the plates. Neglect fringing.

6. (*a*) The footnote on page 816 indicates that Kirchhoff's junction rule is valid at a capacitor plate, and so the conduction current is the same value as the displacement current. Thus the maximum conduction current is $\boxed{35\,\mu\mathrm{A}}$.

(*b*) The charge on the pages is given by $Q = CV = C\mathscr{E}_0 \cos \omega t$. The current is the derivative of this.

$$I = \frac{dQ}{dt} = -\omega C\mathscr{E}_0 \sin \omega t \ ; \ I_{max} = \omega C\mathscr{E}_0 \ \rightarrow$$

$$\mathscr{E}_0 = \frac{I_{max}}{\omega C} = \frac{I_{max} d}{2\pi f \varepsilon_0 A} = \frac{\left(35\times10^{-6}\ \mathrm{A}\right)\left(1.6\times10^{-3}\ \mathrm{m}\right)}{2\pi(76.0\ \mathrm{Hz})\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\pi(0.025\mathrm{m})^2}$$

$$= 6749\ \mathrm{V} \approx \boxed{6700\ \mathrm{V}}$$

(*c*) From Eq. 31-3, $I_D = \varepsilon_0 \dfrac{d\Phi_E}{dt}$.

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} \ \rightarrow \ \left(\frac{d\Phi_E}{dt}\right)_{max} = \frac{(I_D)_{max}}{\varepsilon_0} = \frac{35\times10^{-6}\ \mathrm{A}}{8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}} = \boxed{4.0\times10^6\ \mathrm{V\bullet m/s}}$$

11. (II) The electric field of a plane EM wave is given by $E_x = E_0 \cos(kz + \omega t)$, $E_y = E_z = 0$. Determine (*a*) the direction of propagation and (*b*) the magnitude and direction of $\vec{\mathbf{B}}$.

11. (*a*) If we write the argument of the cosine function as $kz + \omega t = k(z + ct)$, we see that the wave is traveling in the $\boxed{-z \text{ direction}}$, or $\boxed{-\hat{\mathbf{k}}}$.

(*b*) $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and to the direction of propagation. At the origin, the electric field is pointing in the positive *x* direction. Since $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ must point in the negative *z* direction, $\vec{\mathbf{B}}$ must point in the the $\boxed{-y \text{ direction}}$, or $\boxed{-\hat{\mathbf{j}}}$. The magnitude of the magnetic field is found from Eq. 31-11 as $B_0 = \boxed{E_0/c}$.

14. (I) (*a*) What is the wavelength of a 25.75×10^9 Hz radar signal? (*b*) What is the frequency of an X-ray with wavelength 0.12 nm?

14. Use Eq. 31-14 to find the wavelength and frequency.

$$(a)\quad c = \lambda f \;\rightarrow\; \lambda = \frac{c}{f} = \frac{(3.000 \times 10^8\ \text{m/s})}{(25.75 \times 10^9\ \text{Hz})} = \boxed{1.165 \times 10^{-2}\ \text{m}}$$

$$(b)\quad c = \lambda f \;\rightarrow\; f = \frac{c}{\lambda} = \frac{(3.00 \times 10^8\ \text{m/s})}{(0.12 \times 10^{-9}\ \text{m})} = \boxed{2.5 \times 10^{18}\ \text{Hz}}$$

18. (II) Pulsed lasers used for science and medicine produce very brief bursts of electromagnetic energy. If the laser light wavelength is 1062 nm (Neodymium–YAG laser), and the pulse lasts for 38 picoseconds, how many wavelengths are found within the laser pulse? How brief would the pulse need to be to fit only one wavelength?

18. The length of the pulse is $\Delta d = c\Delta t$. Use this to find the number of wavelengths in a pulse.

$$N = \frac{(c\Delta t)}{\lambda} = \frac{\left(3.00\times10^{8}\text{ m/s}\right)\left(38\times10^{-12}\text{s}\right)}{\left(1062\times10^{-9}\text{m}\right)} = 10734 \approx \boxed{11{,}000\text{ wavelengths}}$$

If the pulse is to be only one wavelength long, then its time duration is the period of the wave, which is the reciprocal of the wavelength.

$$T = \frac{1}{f} = \frac{\lambda}{c} = \frac{\left(1062\times10^{-9}\text{m}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)} = \boxed{3.54\times10^{-15}\text{s}}$$

22. (I) The $\vec{\mathbf{E}}$ field in an EM wave has a peak of 26.5 mV/m. What is the average rate at which this wave carries energy across unit area per unit time?

22. The average energy transferred across unit area per unit time is the average magnitude of the Poynting vector, and is given by Eq. 31-19a.

$$\overline{S} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(3.00\times10^{8}\text{ m/s}\right)\left(0.0265\text{ V/m}\right) = \boxed{9.32\times10^{-7}\text{ W/m}^2}$$

28. (II) A 15.8-mW laser puts out a narrow beam 2.00 mm in diameter. What are the rms values of E and B in the beam?

28. The power output per unit area is the intensity, and also is the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{0.0158\,\text{W}}{\pi(1.00\times10^{-3}\,\text{m})^2(3.00\times10^8\,\text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}}$$

$$= 1376.3\,\text{V/m} \approx \boxed{1380\,\text{V/m}}$$

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{1376.3\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{4.59\times10^{-6}\,\text{T}}$$

44. Who will hear the voice of a singer first: a person in the balcony 50.0 m away from the stage (see Fig. 31–24), or a person 1500 km away at home whose ear is next to the radio listening to a live broadcast? Roughly how much sooner? Assume the microphone is a few centimeters from the singer and the temperature is 20°C.

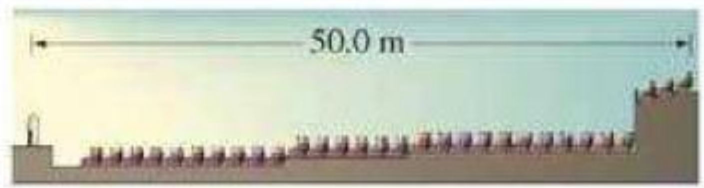

FIGURE 31–24 Problem 44.

44. We ignore the time for the sound to travel to the microphone. Find the difference between the time for sound to travel to the balcony and for a radio wave to travel 3000 km.

$$\Delta t = t_{\text{radio}} - t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) - \left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right) = \left(\frac{3\times10^6\,\text{m}}{3.00\times10^8\,\text{m/s}}\right) - \left(\frac{50\,\text{m}}{343\,\text{m/s}}\right) = -0.14\,\text{s},$$

so the $\boxed{\text{person at the radio hears the voice 0.14 s sooner.}}$

49. What are E_0 and B_0 2.00 m from a 75-W light source? Assume the bulb emits radiation of a single frequency uniformly in all directions.

49. The light has the same intensity in all directions, so use a spherical geometry centered on the source to find the value of the Poynting vector. Then use Eq. 31-19a to find the magnitude of the electric field, and Eq. 31-11 with $v = c$ to find the magnitude of the magnetic field.

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0^2 \quad \rightarrow$$

$$E_0 = \sqrt{\frac{P_0}{2\pi r^2 c\varepsilon_0}} = \sqrt{\frac{(75\,\text{W})}{2\pi(2.00\,\text{m})^2(3.00\times10^8\,\text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2)}} = 33.53\,\text{V/m}$$

$$\approx \boxed{34\,\text{V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{(33.53\,\text{V/m})}{(3.00\times10^8\,\text{m/s})} = \boxed{1.1\times10^{-7}\,\text{T}}$$

55. A point source emits light energy uniformly in all directions at an average rate P_0 with a single frequency f. Show that the peak electric field in the wave is given by

$$E_0 = \sqrt{\frac{\mu_0 c P_0}{2\pi r^2}}.$$

55. The light has the same intensity in all directions. Use a spherical geometry centered at the source with the definition of the Poynting vector.

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0^{\ 2} = \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 \quad \rightarrow \quad \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 = \frac{P_0}{4\pi r^2} \quad \rightarrow \quad \boxed{E_0 = \sqrt{\frac{\mu_0 c P_0}{2\pi r^2}}}$$

58. In free space ("vacuum"), where the net charge and current flow is zero, the speed of an EM wave is given by $v = 1/\sqrt{\epsilon_0 \mu_0}$. If, instead, an EM wave travels in a nonconducting ("dielectric") material with dielectric constant K, then $v = 1/\sqrt{K\epsilon_0 \mu_0}$. For frequencies corresponding to the visible spectrum (near 5×10^{14} Hz), the dielectric constant of water is 1.77. Predict the speed of light in water and compare this value (as a percentage) with the speed of light in a vacuum.

58. We calculate the speed of light in water according to the relationship given.

$$v_{\text{water}} = \frac{1}{\sqrt{K\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}\frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}c = \frac{1}{\sqrt{1.77}}\left(3.00\times10^{8}\text{ m/s}\right) = \boxed{2.25\times10^{8}\text{ m/s}}$$

$$\frac{v_{\text{water}}}{c} = \frac{\frac{1}{\sqrt{K}}c}{c} = \frac{1}{\sqrt{K}} = 0.752 = \boxed{75.2\%}$$

20. (II) An electromagnetic wave has an electric field given by

$$\vec{\mathbf{E}} = \hat{\mathbf{i}}(225\text{ V/m})\sin\left[(0.077\text{ m}^{-1})z - (2.3\times10^{7}\text{ rad/s})t\right].$$

(*a*) What are the wavelength and frequency of the wave? (*b*) Write down an expression for the magnetic field.

20. (*a*) The general form of a plane wave is given in Eq. 31-7. For this wave, $E_x = E_0\sin(kz - \omega t)$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.077\text{m}^{-1}} = 81.60\text{ m} \approx \boxed{82\text{ m}}$$

$$f = \frac{\omega}{2\pi} = \frac{2.3\times10^{7}\text{ rad/s}}{2\pi} = 3.661\times10^{6}\text{ Hz} \approx \boxed{3.7\text{ MHz}}$$

Note that $\lambda f = (81.60\text{ m})(3.661\times10^{6}\text{ Hz}) = 2.987\times10^{8}\text{ m/s} \approx c.$

20. (II) An electromagnetic wave has an electric field given by

$$\vec{\mathbf{E}} = \hat{\mathbf{i}}(225\ \mathrm{V/m})\sin[(0.077\ \mathrm{m^{-1}})z - (2.3 \times 10^7\ \mathrm{rad/s})t].$$

(*a*) What are the wavelength and frequency of the wave? (*b*) Write down an expression for the magnetic field.

(*b*) The magnitude of the magnetic field is given by $B_0 = E_0/c$. The wave is traveling in the $\hat{\mathbf{k}}$ direction, and so the magnetic field must be in the $\hat{\mathbf{j}}$ direction, since the direction of travel is given by the direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$.

$$B_0 = \frac{E_0}{c} = \frac{225\ \mathrm{V/m}}{3.00 \times 10^8\ \mathrm{m/s}} = 7.50 \times 10^{-7}\ \mathrm{T} \quad \rightarrow$$

$$\vec{\mathbf{B}} = \boxed{\hat{\mathbf{j}}\left(7.50 \times 10^{-7}\ \mathrm{T}\right)\sin\left[\left(0.077\mathrm{m^{-1}}\right)z - \left(2.3 \times 10^7\ \mathrm{rad/s}\right)t\right]}$$