1. Radio waves and light are both electromagnetic waves. Why can a radio receive a signal behind a hill when we cannot see the transmitting antenna?

1. Radio waves have a much longer wavelength than visible light and will diffract around normal-sized objects (like hills). The wavelengths of visible light are very small and will not diffract around normal-sized objects.

4. For diffraction by a single slit, what is the effect of increasing (*a*) the slit width, (*b*) the wavelength?

4. (*a*) If the slit width is increased, the diffraction pattern will become more compact.
(*b*) If the wavelength of the light is increased, the diffraction pattern will spread out.

***10.** When both diffraction and interference are taken into account in the double-slit experiment, discuss the effect of increasing (*a*) the wavelength, (*b*) the slit separation, (*c*) the slit width.

10. (*a*) Increasing the wavelength, λ, will spread out the diffraction pattern, since the locations of the minima are given by $\sin\theta = m\lambda/D$. The interference pattern will also spread out; the interference maxima are given by $\sin\theta = m\lambda/d$. The number of interference fringes in the central diffraction maximum will not change.
(*b*) Increasing the slit separation, d, will decrease the spacing between the interference fringes without changing the diffraction, so more interference maxima will fit in the central maximum of the diffraction envelope.
(*c*) Increasing the slit width, D, will decrease the angular width of the diffraction central maximum without changing the interference fringes, so fewer bright fringes will fit in the central maximum.

20. Explain why there are tiny peaks between the main peaks produced by a diffraction grating illuminated with monochromatic light. Why are the peaks so tiny?

20. The tiny peaks are produced when light from some but not all of the slits interferes constructively. The peaks are tiny because light from only some of the slits interferes constructively.

* **23.** What would be the color of the sky if the Earth had no atmosphere?

23. Black. If there were no atmosphere, there would be no scattering of the sunlight coming to Earth.

2. (I) Monochromatic light falls on a slit that is 2.60×10^{-3} mm wide. If the angle between the first dark fringes on either side of the central maximum is 32.0° (dark fringe to dark fringe), what is the wavelength of the light used?

2. The angle from the central maximum to the first dark fringe is equal to half the width of the central maximum. Using this angle and Eq. 35-1, we calculate the wavelength used.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(32^\circ) = 16^\circ$$

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \lambda = D\sin\theta_1 = \left(2.60\times10^{-3}\text{ mm}\right)\sin\left(16^\circ\right) = 7.17\times10^{-4}\text{ mm} = \boxed{717\text{ nm}}$$

7. (II) If a slit diffracts 580-nm light so that the diffraction maximum is 6.0 cm wide on a screen 2.20 m away, what will be the width of the diffraction maximum for light with a wavelength of 460 nm?

7. We use the distance to the screen and half the width of the diffraction maximum to calculate the angular distance to the first minimum. Then using this angle and Eq. 35-1 we calculate the slit width. Then using the slit width and the new wavelength we calculate the angle to the first minimum and the width of the diffraction maximum.

$$\tan\theta_1 = \frac{\left(\frac{1}{2}\Delta y_1\right)}{\ell} \rightarrow \theta_1 = \tan^{-1}\frac{\left(\frac{1}{2}\Delta y_1\right)}{\ell} = \tan^{-1}\frac{\left(\frac{1}{2}\times 0.06\,\text{m}\right)}{2.20\,\text{m}} = 0.781°$$

$$\sin\theta_1 = \frac{\lambda_1}{D} \rightarrow D = \frac{\lambda_1}{\sin\theta_1} = \frac{580\,\text{nm}}{\sin 0.781°} = 42{,}537\,\text{nm}$$

11. (II) Coherent light from a laser diode is emitted through a rectangular area 3.0 μm × 1.5 μm (horizontal-by-vertical). If the laser light has a wavelength of 780 nm, determine the angle between the first diffraction minima (*a*) above and below the central maximum, (*b*) to the left and right of the central maximum.

11. (*a*) For vertical diffraction we use the height of the slit (1.5 μm) as the slit width in Eq. 35-1 to calculate the angle between the central maximum to the first minimum. The angular separation of the first minima is equal to twice this angle.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{1.5\times10^{-6}\,\text{m}} = 31.3°$$

$$\Delta\theta = 2\theta_1 = 2(31.3°) \approx \boxed{63°}$$

(*b*) To find the horizontal diffraction we use the width of the slit (3.0 μm) in Eq. 35-1.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{3.0\times10^{-6}\,\text{m}} = 15.07°$$

$$\Delta\theta = 2\theta_1 = 2(15.07°) \approx \boxed{30°}$$

*18. (II) In a double-slit experiment, if the central diffraction peak contains 13 interference fringes, how many fringes are contained within each secondary diffraction peak (between $m = +1$ and $+2$ in Eq. 35–2). Assume the first diffraction minimum occurs at an interference minimum.

18. In a double-slit experiment, if the central diffraction peak contains 13 interference fringes, there is the $m=0$ fringe, along with fringes up to $m=6$ on each side of $\theta=0$. Then, at angle θ, the $m=7$ interference fringe coincides with the first diffraction minima. We set this angle in Eq. 34-2a and 35-2 equal to solve for the relationship between the slit width and separation.

$$\sin\theta_1 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow \frac{d}{D} = \frac{m}{m'} = \frac{7}{1} = 7 \rightarrow d = 7D$$

Now, we use these equations again to find the m value at the second diffraction minimum, $m'=0$.

$$\sin\theta_2 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow m = m'\frac{d}{D} = 2\frac{7D}{D} = 14$$

Thus, the $\boxed{\text{six fringes}}$ corresponding to $m=8$ to $m=13$ will occur within the first and second diffraction minima.

25. (II) Two stars 16 light-years away are barely resolved by a 66-cm (mirror diameter) telescope. How far apart are the stars? Assume $\lambda = 550\,\text{nm}$ and that the resolution is limited by diffraction.

$\boxed{25.}$ The angular resolution is given by Eq. 35-10. The distance between the stars is the angular resolution times the distance to the stars from the Earth.

$$\theta = 1.22\frac{\lambda}{D} \;;\; \ell = r\theta = 1.22\frac{r\lambda}{D} = 1.22\frac{(16\,\text{ly})\left(\frac{9.46\times10^{15}\,\text{m}}{1\,\text{ly}}\right)(550\times10^{-9}\,\text{m})}{(0.66\,\text{m})} = \boxed{1.5\times10^{11}\,\text{m}}$$

29. (II) The normal lens on a 35-mm camera has a focal length of 50.0 mm. Its aperture diameter varies from a maximum of 25 mm $(f/2)$ to a minimum of 3.0 mm $(f/16)$. Determine the resolution limit set by diffraction for $(f/2)$ and $(f/16)$. Specify as the number of lines per millimeter resolved on the detector or film. Take $\lambda = 560$ nm.

29. We set the resolving power as the focal length of the lens multiplied by the angular resolution, as in Eq. 35-11. The resolution is the inverse of the resolving power.

$$\frac{1}{RP(f/2)} = \left[\frac{1.22\lambda f}{D}\right]^{-1} = \frac{D}{1.22\lambda f} = \frac{25\text{ mm}}{1.22\left(560\times10^{-6}\text{mm}\right)(50.0\text{ mm})} = \boxed{730\text{ lines/mm}}$$

$$\frac{1}{RP(f/16)} = \frac{3.0\text{ mm}}{1.22\left(560\times10^{-6}\text{mm}\right)(50.0\text{ mm})} = \boxed{88\text{ lines/mm}}$$

35. (II) Red laser light from a He–Ne laser $(\lambda = 632.8\text{ nm})$ is used to calibrate a diffraction grating. If this light creates a second-order fringe at 53.2° after passing through the grating, and light of an unknown wavelength λ creates a first-order fringe at 20.6°, find λ.

35. Since the same diffraction grating is being used for both wavelengths of light, the slit separation will be the same. We solve Eq. 35-13 for the slit separation for both wavelengths and set the two equations equal. The resulting equation is then solved for the unknown wavelength.

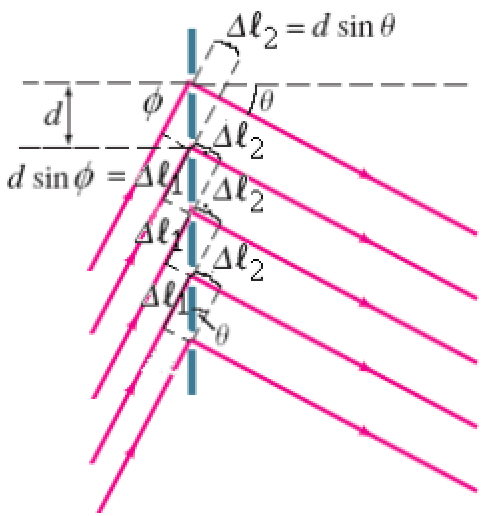
$$d\sin\theta = m\lambda \Rightarrow d = \frac{m_1\lambda_1}{\sin\theta_1} = \frac{m_2\lambda_2}{\sin\theta_2} \Rightarrow \lambda_2 = \frac{m_1}{m_2}\frac{\sin\theta_2}{\sin\theta_1}\lambda_1 = \frac{2}{1}\frac{\sin 20.6^\circ}{\sin 53.2^\circ}(632.8\text{ nm}) = \boxed{556\text{ nm}}$$

45. (II) Monochromatic light falls on a transmission diffraction grating at an angle ϕ to the normal. (*a*) Show that Eq. 35–13 for diffraction maxima must be replaced by

$$d(\sin\phi + \sin\theta) = \pm m\lambda. \qquad m = 0, 1, 2, \cdots.$$

(*b*) Explain the $\pm$ sign. (*c*) Green light with a wavelength of 550 nm is incident at an angle of 15° to the normal on a diffraction grating with 5000 lines/cm. Find the angles at which the first-order maxima occur.

45. (*a*) Diffraction maxima occur at angles for which the incident light constructive interferes. That is, when the path length difference between two rays is equal to an integer number of wavelengths. Since the light is incident at an angle ϕ relative to the grating, each succeeding higher ray, as shown in the diagram, travels a distance $\Delta\ell_1 = d\sin\phi$ farther to reach the grating. After passing through the grating the higher rays travel a distance to the screen that is again longer by $\Delta\ell_2 = d\sin\theta$. By setting the total path length difference equal to an integer number of wavelengths, we are able to determine the location of the bright fringes.



$$\Delta\ell = \Delta\ell_1 + \Delta\ell_1 = d\left(\sin\phi + \sin\theta\right) = \pm m\lambda, \qquad m = 0,1,2,....$$

(*b*) The $\pm$ allows for the incident angle and the diffracted angle to have positive and negative values.

(*c*) We insert the given data, with m=1, to solve for the angles θ.

$$\theta = \sin^{-1}\left(-\sin\phi \pm \frac{m\lambda}{d}\right) = \sin^{-1}\left(-\sin 15^\circ \pm \frac{550\times10^{-9}\,\text{m}}{0.01\,\text{m}/5000\,\text{lines}}\right) = \boxed{0.93^\circ \text{ and } -32^\circ}$$

*48. (II) Let 580-nm light be incident normally on a diffraction grating for which $d = 3.00D = 1050\,\text{nm}$. (*a*) How many orders (principal maxima) are present? (*b*) If the grating is 1.80 cm wide, what is the full angular width of each principal maximum?

48. (*a*) We use Eq. 35-13, with the angle equal to 90° to determine the maximum order.

$$\sin\theta = \frac{m\lambda}{d} \quad\rightarrow\quad m = \frac{d\sin\theta}{\lambda} = \frac{(1050\,\text{nm})\sin 90^\circ}{580\,\text{nm}} = 1.81$$

Since the order must be an integer number there will only be one principal maximum on either side of the central maximum. Counting the central maximum and the two other principal maxima there will be a total of three principal maxima.

(*b*) We use Eq. 35-17 to calculate the peak width, where the full peak width is double the half-peak width and the angle to the peak is given by Eq. 35-13.

$$\theta_0 = 0$$

$$\Delta\theta_0 = 2\frac{\lambda}{Nd\cos\theta_0} = \frac{2\lambda}{\ell\cos\theta_0} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos 0^\circ} = 6.4\times10^{-5}\,\text{rad} = \boxed{0.0037^\circ}$$

$$\theta_{\pm1} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{\pm1\times580\,\text{nm}}{1050\,\text{nm}}\right) = \pm33.5^\circ$$

$$\Delta\theta_{\pm1} = \frac{2\lambda}{\ell\cos\theta_{\pm1}} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos(\pm33.5^\circ)} = 7.7\times10^{-5}\,\text{rad} = \boxed{0.0044^\circ}$$

52. (I) Two polarizers are oriented at 65° to one another. Unpolarized light falls on them. What fraction of the light intensity is transmitted?

52. Use Eq. 35-21. Since the initial light is unpolarized, the intensity after the first polarizer will be half the initial intensity. Let the initial intensity be I_0.

$$I_1 = \tfrac{1}{2} I_0 \ ; \ I_2 = I_1 \cos^2 \theta = \tfrac{1}{2} I_0 \cos^2 \theta \rightarrow \frac{I_2}{I_0} = \frac{\cos^2 65°}{2} = \boxed{0.089}$$

56. (II) The critical angle for total internal reflection at a boundary between two materials is 55°. What is Brewster's angle at this boundary? Give two answers, one for each material.

56. The critical angle exists when light passes from a material with a higher index of refraction (n_1) into a material with a lower index of refraction (n_2). Use Eq. 32-7.

$$\frac{n_2}{n_1} = \sin \theta_C = \sin 55°$$

To find the Brewster angle, use Eq. 35-22a. If light is passing from high index to low index, we have the following.

$$\frac{n_2}{n_1} = \tan \theta_p = \sin 55° \rightarrow \theta_p = \tan^{-1}(\sin 55°) = \boxed{39°}$$

If light is passing from low index to high index, we have the following.

$$\frac{n_1}{n_2} = \tan \theta_p = \frac{1}{\sin 55°} \rightarrow \theta_p = \tan^{-1}\left(\frac{1}{\sin 55°}\right) = \boxed{51°}$$

61. (II) Two polarizers A and B are aligned so that their transmission axes are vertical and horizontal, respectively. A third polarizer is placed between these two with its axis aligned at angle θ with respect to the vertical. Assuming vertically polarized light of intensity I_0 is incident upon polarizer A, find an expression for the light intensity I transmitted through this three-polarizer sequence. Calculate the derivative $dI/d\theta$; then use it to find the angle θ that maximizes I.

61. We assume vertically polarized light of intensity I_0 is incident upon the first polarizer. The angle between the polarization direction and the polarizer is θ. After the light passes that first polarizer, the angle between that light and the next polarizer will be $90° - \theta$. Apply Eq. 35-21.

$$I_1 = I_0 \cos^2\theta \ ; \ I = I_1 \cos^2(90° - \theta) = I_0 \cos^2\theta \cos^2(90° - \theta) = \boxed{I_0 \cos^2\theta \sin^2\theta}$$

We can also use the trigonometric identity $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ to write the final intensity as

$$I = I_0 \cos^2\theta \sin^2\theta = \boxed{\tfrac{1}{4} I_0 \sin^2 2\theta}.$$

$$\frac{dI}{d\theta} = \frac{d}{d\theta}\left(\tfrac{1}{4} I_0 \sin^2 2\theta\right) = \tfrac{1}{4} I_0 (2\sin 2\theta)(\cos 2\theta)2 = I_0 \sin 2\theta \cos 2\theta = \boxed{\tfrac{1}{2} I_0 \sin 4\theta}$$

$$\tfrac{1}{2} I_0 \sin 4\theta = 0 \ \rightarrow \ 4\theta = 0, 180°, 360° \ \rightarrow \ \theta = 0, 45°, 90°$$

Substituting the three angles back into the intensity equation, we see that the angles $0°$ and $90°$ both give minimum intensity. The angle $\boxed{45° \text{ gives the maximum intensity}}$ of $\frac{1}{4} I_0$.

67. Light is incident on a diffraction grating with 7600 lines/cm and the pattern is viewed on a screen located 2.5 m from the grating. The incident light beam consists of two wavelengths, $\lambda_1 = 4.4 \times 10^{-7}$ m and $\lambda_2 = 6.8 \times 10^{-7}$ m. Calculate the linear distance between the first-order bright fringes of these two wavelengths on the screen.

67. We find the angles for the first order from Eq. 35-13.

$$\theta_1 = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(4.4\times10^{-7}\ \text{m})}{0.01\ \text{m}/7600} = 19.5^\circ$$

$$\theta_2 = \sin^{-1}\frac{(1)(6.8\times10^{-7}\ \text{m})}{0.01\ \text{m}/7600} = 31.1^\circ$$

The distances from the central white line on the screen are found using the tangent of the angle and the distance to the screen.

$$y_1 = L\tan\theta_1 = (2.5\,\text{m})\tan 19.5^\circ = 0.89\,\text{m}$$

$$y_2 = L\tan\theta_2 = (2.5\,\text{m})\tan 31.1^\circ = 1.51\,\text{m}$$

Subtracting these two distances gives the linear separation of the two lines.

$$y_2 - y_1 = 1.51\,\text{m} - 0.89\,\text{m} = \boxed{0.6\,\text{m}}$$