1. If the electric flux through a closed surface is zero, is the electric field necessarily zero at all points on the surface? Explain. What about the converse: If $\vec{\mathbf{E}} = 0$ at all points on the surface is the flux through the surface zero?

2. Is the electric field $\vec{\mathbf{E}}$ in Gauss's law, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q_{\text{encl}}/\epsilon_0$, created only by the charge Q_{encl}?

1. No. If the net electric flux through a surface is zero, then the net charge contained in the surface is zero. However, there may be charges both inside and outside the surface that affect the electric field at the surface. The electric field could point outward from the surface at some points and inward at others. Yes. If the electric field is zero for all points on the surface, then the net flux through the surface must be zero and no net charge is contained within the surface.

2. No. The electric field in the expression for Gauss's law refers to the *total* electric field, not just the electric field due to any enclosed charge. Notice, though, that if the electric field is due to a charge outside the Gaussian surface, then the net flux through the surface due to this charge will be zero.

12. A solid conductor carries a net positive charge Q. There is a hollow cavity within the conductor, at whose center is a negative point charge $-q$ (Fig. 22–24). What is the charge on (*a*) the outer surface of the conductor and (*b*) the inner surface of the conductor's cavity?

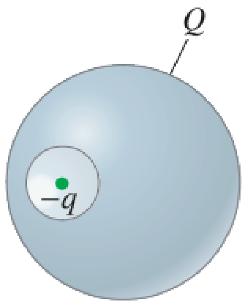


FIGURE 22–24
Question 12.

12. (*a*) A charge of $(Q - q)$ will be on the outer surface of the conductor. The total charge Q is placed on the conductor but since $+q$ will reside on the inner surface, the leftover, $(Q - q)$, will reside on the outer surface.
(*b*) A charge of $+q$ will reside on the inner surface of the conductor since that amount is attracted by the charge $-q$ in the cavity. (Note that E must be zero inside the conductor.)

3. (II) A cube of side ℓ is placed in a uniform field E_0 with edges parallel to the field lines. (*a*) What is the net flux through the cube? (*b*) What is the flux through each of its six faces?

3. (*a*) Since the field is uniform, no lines originate or terminate inside the cube, and so the net flux is $\Phi_{\text{net}} = \boxed{0}$.
(*b*) There are two opposite faces with field lines perpendicular to the faces. The other four faces have field lines parallel to those faces. For the faces parallel to the field lines, no field lines enter or exit the faces. Thus $\Phi_{\text{parallel}} = \boxed{0}$.

Of the two faces that are perpendicular to the field lines, one will have field lines entering the cube, and so the angle between the field lines and the face area vector is 180°. The other will have field lines exiting the cube, and so the angle between the field lines and the face area vector is 0°. Thus we have $\Phi_{\text{entering}} = \vec{E} \bullet \vec{A} = E_0 A \cos 180° = \boxed{-E_0 \ell^2}$ and $\Phi_{\text{leaving}} = \vec{E} \bullet \vec{A} = E_0 A \cos 0° = \boxed{E_0 \ell^2}$.

7. (II) In Fig. 22–27, two objects, O_1 and O_2, have charges $+1.0\,\mu C$ and $-2.0\,\mu C$ respectively, and a third object, O_3, is electrically neutral. (*a*) What is the electric flux through the surface A_1 that encloses all the three objects? (*b*) What is the electric flux through the surface A_2 that encloses the third object only?

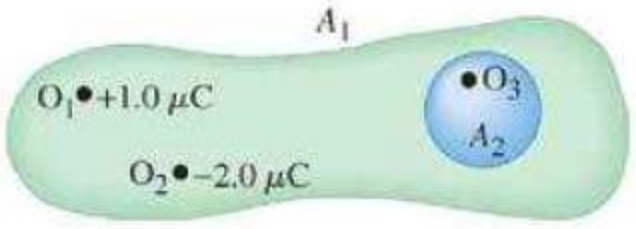


FIGURE 22–27 Problem 7.

7. (*a*) Use Gauss's law to determine the electric flux.

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_o} = \frac{-1.0\times10^{-6}\,C}{8.85\times10^{-12}\,C^2/N\cdot m^2} = \boxed{-1.1\times10^{5}\,N\cdot m^2/C}$$

(*b*) Since there is no charge enclosed by surface A_2, $\Phi_E = \boxed{0}$.

9. (II) In a certain region of space, the electric field is constant in direction (say horizontal, in the x direction), but its magnitude decreases from $E = 560\,N/C$ at $x = 0$ to $E = 410\,N/C$ at $x = 25\,m$. Determine the charge within a cubical box of side $\ell = 25\,m$, where the box is oriented so that four of its sides are parallel to the field lines (Fig. 22–28).

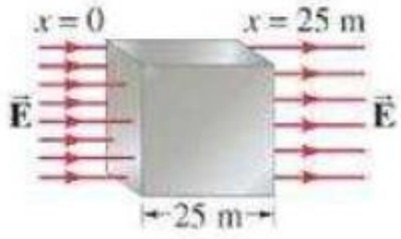


FIGURE 22–28 Problem 9.

9. The only contributions to the flux are from the faces perpendicular to the electric field. Over each of these two surfaces, the magnitude of the field is constant, so the flux is just $\vec{\mathbf{E}}\cdot\vec{\mathbf{A}}$ on each of these two surfaces.

$$\Phi_E = \left(\vec{\mathbf{E}}\cdot\vec{\mathbf{A}}\right)_{right} + \left(\vec{\mathbf{E}}\cdot\vec{\mathbf{A}}\right)_{left} = E_{right}\ell^2 - E_{left}\ell^2 = \frac{Q_{encl}}{\varepsilon_0} \rightarrow$$

$$Q_{encl} = \left(E_{right} - E_{left}\right)\ell^2\varepsilon_0 = (410\,N/C - 560\,N/C)(25\,m)^2\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right) = \boxed{-8.3\times10^{-7}\,C}$$

24. (II) Two large, flat metal plates are separated by a distance that is very small compared to their height and width. The conductors are given equal but opposite uniform surface charge densities $\pm\sigma$. Ignore edge effects and use Gauss's law to show (*a*) that for points far from the edges, the electric field between the plates is $E = \sigma/\epsilon_0$ and (*b*) that outside the plates on either side the field is zero. (*c*) How would your results be altered if the two plates were nonconductors? (See Fig. 22–30).

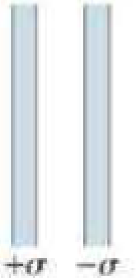
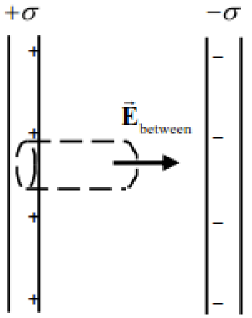


FIGURE 22–30
Problems 24, 25, and 26.

24. Since the charges are of opposite sign, and since the charges are free to move since they are on conductors, the charges will attract each other and move to the inside or facing edges of the plates. There will be no charge on the outside edges of the plates. And there cannot be charge in the plates themselves, since they are conductors. All of the charge must reside on surfaces. Due to the symmetry of the problem, all field lines must be perpendicular to the plates, as discussed in Example 22-7.

(*a*) To find the field between the plates, we choose a gaussian cylinder, perpendicular to the plates, with area A for the ends of the cylinder. We place one end inside the left plate (where the field must be zero), and the other end between the plates. No flux passes through the curved surface of the cylinder.

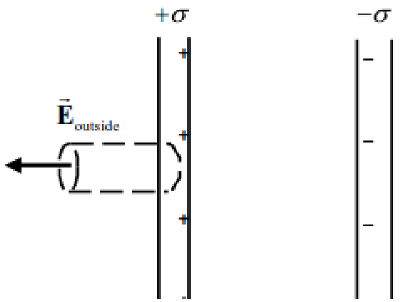


$$\oint \vec{E}\cdot d\vec{A} = \int_{ends} \vec{E}\cdot d\vec{A} + \int_{side} \vec{E}\cdot d\vec{A} = \int_{\substack{right\\end}} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \rightarrow$$

$$E_{between} A = \frac{\sigma A}{\varepsilon_0} \rightarrow \boxed{E_{between} = \frac{\sigma}{\varepsilon_0}}$$

The field lines between the plates leave the inside surface of the left plate, and terminate on the inside surface of the right plate. A similar derivation could have been done with the right end of the cylinder inside of the right plate, and the left end of the cylinder in the space between the plates.

(*b*) If we now put the cylinder from above so that the right end is inside the conducting material, and the left end is to the left of the left plate, the only possible location for flux is through the left end of the cylinder. Note that there is NO charge enclosed by the Gaussian cylinder.



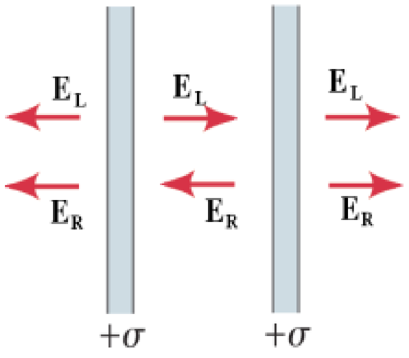
$$\oint \vec{E}\cdot d\vec{A} = \int_{ends} \vec{E}\cdot d\vec{A} + \int_{side} \vec{E}\cdot d\vec{A} = \int_{\substack{left\\end}} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \rightarrow$$

$$E_{outside} A = \frac{0}{\varepsilon_0} \rightarrow \boxed{E_{outside} = \frac{0}{\varepsilon_0}}$$

(*c*) If the two plates were nonconductors, the results would not change. The charge would be distributed over the two plates in a different fashion, and the field inside of the plates would not be zero, but the charge in the empty regions of space would be the same as when the plates are conductors.

25. (II) Suppose the two conducting plates in Problem 24 have the *same* sign and magnitude of charge. What then will be the electric field (*a*) between them and (*b*) outside them on either side? (*c*) What if the plates are nonconducting?

25. Example 22-7 gives the electric field from a positively charged plate as $E = \sigma / 2\varepsilon_0$ with the field pointing away from the plate. The fields from the two plates will add, as shown in the figure.



(*a*) Between the plates the fields are equal in magnitude, but point in opposite directions.

$$E_{\text{between}} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \boxed{0}$$

(*b*) Outside the two plates the fields are equal in magnitude and point in the same direction.

$$E_{\text{outside}} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

(*c*) When the plates are conducting the charge lies on the surface of the plates. For nonconducting plates the same charge will be spread across the plate. This will not affect the electric field between or outside the two plates. It will, however, allow for a non-zero field inside each plate.

42. (II) An uncharged solid conducting sphere of radius r_0 contains two spherical cavities of radii r_1 and r_2, respectively. Point charge Q_1 is then placed within the cavity of radius r_1 and point charge Q_2 is placed within the cavity of radius r_2 (Fig. 22–38). Determine the resulting electric field (magnitude and direction) at locations outside the solid sphere ($r > r_0$), where r is the distance from its center.

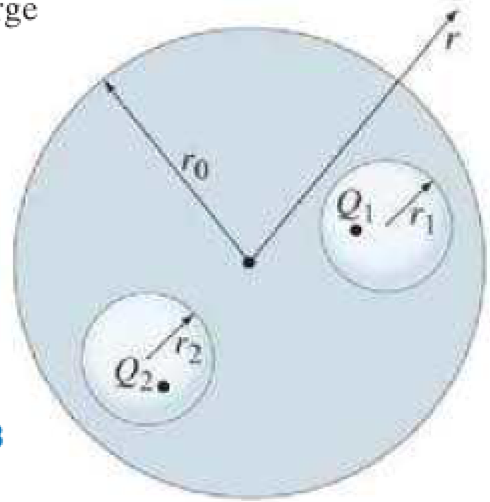


FIGURE 22–38 Problem 42.

42. The conducting sphere is uncharged, and the electric field is 0 everywhere within its interior, except for in the cavities. When charge Q_1 is placed in the first cavity, a charge $-Q_1$ will be drawn from the conducting material to the inner surface of the cavity, and the electric field will remain 0 in the conductor. That charge $-Q_1$ will NOT be distributed symmetrically on the cavity surface. Since the conductor is neutral, a compensating charge Q_1 will appear on the outer surface of the conductor (charge can only be on the surfaces of conductors in electrostatics). Likewise, when charge Q_2 is placed in the second cavity, a charge $-Q_2$ will be drawn from the conducting material, and a compensating charge Q_2 will appear on the outer surface. Since there is no electric field in the conducting material, there is no way for the charges in the cavities to influence the charge distribution on the outer surface. So the distribution of charge on the outer surface is uniform, just as it would be if there were no inner charges, and instead a charge $Q_1 + Q_2$ were simply placed on the conductor. Thus the field outside the conductor is due to a spherically symmetric distribution of $Q_1 + Q_2$. Application of Gauss's law leads to $E = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q_1 + Q_2}{r^2}}$. If $Q_1 + Q_2 > 0$, the field will point radially outward. If $Q_1 + Q_2 < 0$, the field will point radially inward.

57. A point charge Q is placed a distance $r_0/2$ above the surface of an imaginary spherical surface of radius r_0 (Fig. 22–43). (*a*) What is the electric flux through the sphere? (*b*) What range of values does E have at the surface of the sphere? (*c*) Is $\vec{\mathbf{E}}$ perpendicular to the sphere at all points? (*d*) Is Gauss's law useful for obtaining E at the surface of the sphere?

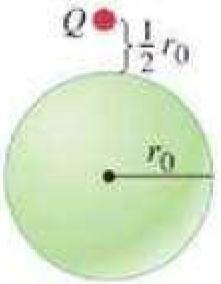
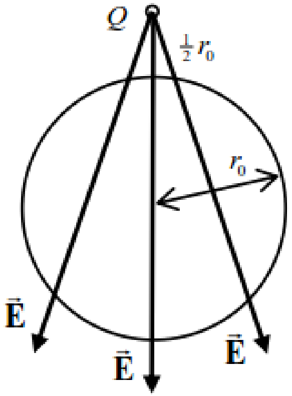


FIGURE 22–43
Problem 57.

57. (*a*) There is no charge enclosed within the sphere, and so no flux lines can originate or terminate inside the sphere. All field lines enter and leave the sphere. Thus the net flux is $\boxed{0}$.

(*b*) The maximum electric field will be at the point on the sphere closest to Q, which is the top of the sphere. The minimum electric field will be at the point on the sphere farthest from Q, which is the bottom of the sphere.



$$E_{\text{max}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_{\text{closest}}^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\frac{1}{2}r_0\right)^2} = \boxed{\frac{1}{\pi\varepsilon_0}\frac{Q}{r_0^2}}$$

$$E_{\text{min}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_{\text{farthest}}^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\frac{5}{2}r_0\right)^2} = \boxed{\frac{1}{25\pi\varepsilon_0}\frac{Q}{r_0^2}}$$

Thus the range of values is $\boxed{\frac{1}{\pi\varepsilon_0}\frac{Q}{r_0^2} \le E_{\substack{\text{sphere}\\ \text{surface}}} \le \frac{1}{25\pi\varepsilon_0}\frac{Q}{r_0^2}}$.

(*c*) $\vec{\mathbf{E}}$ is $\boxed{\text{not perpendicular}}$ at all points. It is only perpendicular at the two points already discussed: the point on the sphere closest to the point charge, and the point on the sphere farthest from the point charge.

(*d*) The electric field is not perpendicular or constant over the surface of the sphere. Therefore Gauss's law is $\boxed{\text{not useful}}$ for obtaining E at the surface of the sphere because a gaussian surface cannot be chosen that simplifies the flux integral.