18. Consider the electric field at the three points indicated by the letters A, B, and C in Fig. 21–51. First draw an arrow at each point indicating the direction of the net force that a positive test charge would experience if placed at that point, then list the letters in order of *decreasing* field strength (strongest first).

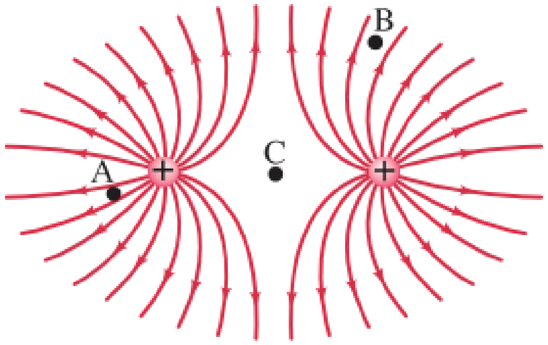


FIGURE 21–51 Question 18.

18. At point C, the positive test charge would experience zero net force. At points A and B, the direction of the force on the positive test charge would be the same as the direction of the field. This direction is indicated by the arrows on the field lines. The strongest field is at point A, followed (in order of decreasing field strength) by B and then C.

19. Why can electric field lines never cross?

19. Electric field lines can never cross because they give the direction of the electrostatic force on a positive test charge. If they were to cross, then the force on a test charge at a given location would be in more than one direction. This is not possible.

24. We wish to determine the electric field at a point near a positively charged metal sphere (a good conductor). We do so by bringing a small test charge, q_0, to this point and measure the force F_0 on it. Will F_0/q_0 be greater than, less than, or equal to the electric field $\vec{\mathbf{E}}$ as it was at that point before the test charge was present?

24. The value measured will be slightly less than the electric field value at that point before the test charge was introduced. The test charge will repel charges on the surface of the conductor and these charges will move along the surface to increase their distances from the test charge. Since they will then be at greater distances from the point being tested, they will contribute a smaller amount to the field.

12. (II) Particles of charge +75, +48, and −85 μC are placed in a line (Fig. 21–52). The center one is 0.35 m from each of the others. Calculate the net force on each charge due to the other two.

FIGURE 21–52 Problem 12.

12. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k = 8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}$.

$$\vec{\mathbf{F}}_{+75} = -k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(75\mu\mathrm{C})(85\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} = -147.2\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-150\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{+48} = k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(48\mu\mathrm{C})(85\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = 563.5\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{560\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{-85} = -k\frac{(85\mu\mathrm{C})(75\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} - k\frac{(85\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = -416.3\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-420\,\mathrm{N}\,\hat{\mathbf{i}}}$$

13. (II) Three charged particles are placed at the corners of an equilateral triangle of side 1.20 m (Fig. 21–53). The charges are +7.0 μC, −8.0 μC, and −6.0 μC. Calculate the magnitude and direction of the net force on each due to the other two.

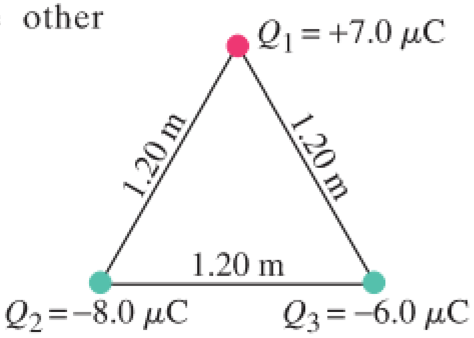
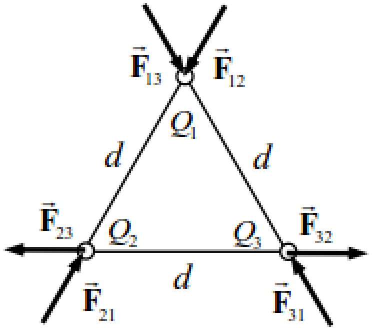


FIGURE 21–53 Problem 13.

13. The forces on each charge lie along a line connecting the charges. Let the variable d represent the length of a side of the triangle. Since the triangle is equilateral, each angle is 60°. First calculate the magnitude of each individual force.



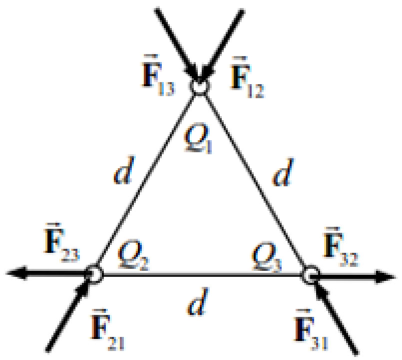
$$F_{12} = k\frac{|Q_1Q_2|}{d^2} = \left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{(7.0\times10^{-6}\,\mathrm{C})(8.0\times10^{-6}\,\mathrm{C})}{(1.20\,\mathrm{m})^2}$$

$$= 0.3495\,\mathrm{N}$$

$$F_{13} = k\frac{|Q_1Q_3|}{d^2} = \left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{(7.0\times10^{-6}\,\mathrm{C})(6.0\times10^{-6}\,\mathrm{C})}{(1.20\,\mathrm{m})^2}$$

$$= 0.2622\,\mathrm{N}$$

$$F_{23} = k\frac{|Q_2Q_3|}{d^2} = \left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{(8.0\times10^{-6}\,\mathrm{C})(6.0\times10^{-6}\,\mathrm{C})}{(1.20\,\mathrm{m})^2} = 0.2996\,\mathrm{N} = F_{32}$$



Now calculate the net force on each charge and the direction of that net force, using components.

$$F_{1x} = F_{12x} + F_{13x} = -(0.3495\,\text{N})\cos 60^\circ + (0.2622\,\text{N})\cos 60^\circ = -4.365\times10^{-2}\,\text{N}$$

$$F_{1y} = F_{12y} + F_{13y} = -(0.3495\,\text{N})\sin 60^\circ - (0.2622\,\text{N})\sin 60^\circ = -5.297\times10^{-1}\,\text{N}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \boxed{0.53\,\text{N}} \qquad \theta_1 = \tan^{-1}\frac{F_{1y}}{F_{1x}} = \tan^{-1}\frac{-5.297\times10^{-1}\,\text{N}}{-4.365\times10^{-2}\,\text{N}} = \boxed{265^\circ}$$

$$F_{2x} = F_{21x} + F_{23x} = (0.3495\,\text{N})\cos 60^\circ - (0.2996\,\text{N}) = -1.249\times10^{-1}\,\text{N}$$

$$F_{2y} = F_{21y} + F_{23y} = (0.3495\,\text{N})\sin 60^\circ + 0 = 3.027\times10^{-1}\,\text{N}$$

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \boxed{0.33\,\text{N}} \qquad \theta_2 = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{3.027\times10^{-1}\,\text{N}}{-1.249\times10^{-1}\,\text{N}} = \boxed{112^\circ}$$

$$F_{3x} = F_{31x} + F_{32x} = -(0.2622\,\text{N})\cos 60^\circ + (0.2996\,\text{N}) = 1.685\times10^{-1}\,\text{N}$$

$$F_{3y} = F_{31y} + F_{32y} = (0.2622\,\text{N})\sin 60^\circ + 0 = 2.271\times10^{-1}\,\text{N}$$

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \boxed{0.26\,\text{N}} \qquad \theta_3 = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{2.271\times10^{-1}\,\text{N}}{1.685\times10^{-1}\,\text{N}} = \boxed{53^\circ}$$

20. (III) Two small charged spheres hang from cords of equal length ℓ as shown in Fig. 21–55 and make small angles θ_1 and θ_2 with the vertical. (*a*) If $Q_1 = Q$, $Q_2 = 2Q$, and $m_1 = m_2 = m$, determine the ratio θ_1/θ_2. (*b*) If $Q_1 = Q$, $Q_2 = 2Q$, $m_1 = m$, and $m_2 = 2m$, determine the ratio θ_1/θ_2. (*c*) Estimate the distance between the spheres for each case.

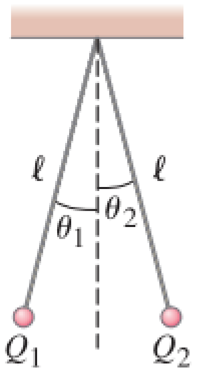
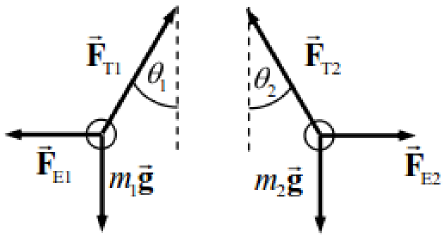


FIGURE 21–55 Problem 20.

20. If all of the angles to the vertical (in both cases) are assumed to be small, then the spheres only have horizontal displacement, and so the electric force of repulsion is always horizontal. Likewise, the small angle condition leads to $\tan\theta \approx \sin\theta \approx \theta$ for all small angles. See the free-body diagram for each sphere, showing the three forces of gravity, tension, and the electrostatic force. Take to the right to be the positive horizontal direction, and up to be the positive vertical direction. Since the spheres are in equilibrium, the net force in each direction is zero.



(*a*) $\sum F_{1x} = F_{T1}\sin\theta_1 - F_{E1} = 0 \;\rightarrow\; F_{E1} = F_{T1}\sin\theta_1$

$$\sum F_{1y} = F_{T1}\cos\theta_1 - m_1 g \;\rightarrow\; F_{T1} = \frac{m_1 g}{\cos\theta_1} \;\rightarrow\; F_{E1} = \frac{m_1 g}{\cos\theta_1}\sin\theta_1 = m_1 g\tan\theta_1 = m_1 g\theta_1$$

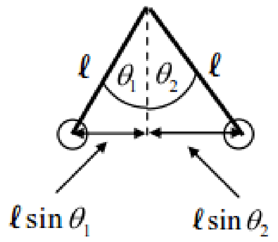
A completely parallel analysis would give $F_{E2} = m_2 g\theta_2$. Since the electric forces are a Newton's third law pair, they can be set equal to each other in magnitude.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_2/m_1 = \boxed{1}$$

(*b*) The same analysis can be done for this case.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_2/m_1 = \boxed{2}$$

(*c*) The horizontal distance from one sphere to the other is s by the small angle approximation. See the diagram. Use the relationship derived above that $F_E = mg\theta$ to solve for the distance.



Case 1: $d = \ell(\theta_1 + \theta_2) = 2\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{d}{2\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{d}{2\ell} \;\rightarrow\; \boxed{d = \left(\frac{4\ell kQ^2}{mg}\right)^{1/3}}$$

Case 2: $d = \ell(\theta_1 + \theta_2) = \dfrac{3}{2}\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{2d}{3\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{2d}{3\ell} \;\rightarrow\; \boxed{d = \left(\frac{3\ell kQ^2}{mg}\right)^{1/3}}$$

25. (I) The electric force on a $+4.20\text{-}\mu C$ charge is $\vec{\mathbf{F}} = (7.22 \times 10^{-4}\,\mathrm{N})\hat{\mathbf{j}}$. What is the electric field at the position of the charge?

26. (I) What is the electric field at a point when the force on a $1.25\text{-}\mu C$ charge placed at that point is $\vec{\mathbf{F}} = (3.0\hat{\mathbf{i}} - 3.9\hat{\mathbf{j}}) \times 10^{-3}\,\mathrm{N}$?

33. (II) Calculate the electric field at one corner of a square 1.22 m on a side if the other three corners are occupied by 2.25×10^{-6} C charges.

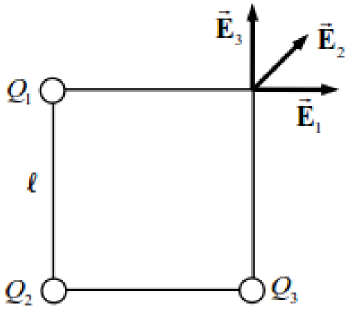
25. Use the definition of the electric field, Eq. 21-3.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{\left(7.22\times10^{-4}\,\mathrm{N}\,\hat{\mathbf{j}}\right)}{4.20\times10^{-6}\,\mathrm{C}} = \boxed{172\,\mathrm{N/C}\,\hat{\mathbf{j}}}$$

26. Use the definition of the electric field, Eq. 21-3.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{\left(3.0\hat{\mathbf{i}} - 3.9\hat{\mathbf{j}}\right)\times10^{-3}\,\mathrm{N}}{1.25\times10^{-6}\,\mathrm{C}} = \boxed{\left(2400\,\hat{\mathbf{i}} - 3100\,\hat{\mathbf{j}}\right)\mathrm{N/C}}$$

33. The field at the upper right corner of the square is the vector sum of the fields due to the other three charges. Let the variable ℓ represent the 1.0 m length of a side of the square, and let the variable Q represent the charge at each of the three occupied corners.



$$E_1 = k\frac{Q}{\ell^2} \;\rightarrow\; E_{1x} = k\frac{Q}{\ell^2}\,,\; E_{1y} = 0$$

$$E_2 = k\frac{Q}{2\ell^2} \;\rightarrow\; E_{2x} = k\frac{Q}{2\ell^2}\cos45^\circ = k\frac{\sqrt{2}Q}{4\ell^2}\,,\; E_{2y} = k\frac{\sqrt{2}Q}{4\ell^2}$$

$$E_3 = k\frac{Q}{\ell^2} \;\rightarrow\; E_{3x} = 0\,,\; E_{1y} = k\frac{Q}{\ell^2}$$

Add the x and y components together to find the total electric field, noting that $E_x = E_y$.

$$E_x = E_{1x} + E_{2x} + E_{3x} = k\frac{Q}{\ell^2} + k\frac{\sqrt{2}Q}{4\ell^2} + 0 = k\frac{Q}{\ell^2}\left(1 + \frac{\sqrt{2}}{4}\right) = E_y$$

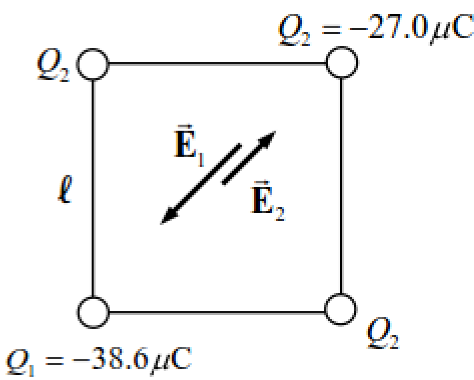
$$E = \sqrt{E_x^2 + E_y^2} = k\frac{Q}{\ell^2}\left(1+\frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q}{\ell^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$= \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(2.25\times10^{-6}\,\mathrm{C}\right)}{\left(1.22\,\mathrm{m}\right)^2}\left(\sqrt{2}+\frac{1}{2}\right) = \boxed{2.60\times10^4\,\mathrm{N/C}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \boxed{45.0^\circ}$$ from the x-direction.

34. (II) Calculate the electric field at the center of a square 52.5 cm on a side if one corner is occupied by a $-38.6\,\mu$C charge and the other three are occupied by $-27.0\,\mu$C charges.

34. The field at the center due to the two -27.0μC negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the other two charges need to be considered. The field due to each of the other charges will point directly toward the charge. Accordingly, the two fields are in opposite directions and can be combined algebraically.



$$E = E_1 - E_2 = k\frac{|Q_1|}{\ell^2/2} - k\frac{|Q_2|}{\ell^2/2} = k\frac{|Q_1| - |Q_2|}{\ell^2/2}$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(38.6-27.0)\times10^{-6}\,\text{C}}{(0.525\,\text{m})^2/2}$$

$$= \boxed{7.57\times10^6\ \text{N/C, towards the } -38.6\mu\text{C charge}}$$

56. (II) An electron with speed $v_0 = 27.5\times10^6\,$m/s is traveling parallel to a uniform electric field of magnitude $E = 11.4\times10^3\,$N/C. (*a*) How far will the electron travel before it stops? (*b*) How much time will elapse before it returns to its starting point?

56. (*a*) Since the field is uniform, the electron will experience a constant force in the direction opposite to its velocity, so the acceleration is constant and negative. Use constant acceleration relationships with a final velocity of 0.

$$F = ma = qE = -eE \ \rightarrow\ a = -\frac{eE}{m}\ ;\ v^2 = v_0^2 + 2a\Delta x = 0 \ \rightarrow$$

$$\Delta x = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2\left(-\frac{eE}{m}\right)} = \frac{mv_0^2}{2eE} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(27.5\times10^6\ \text{m/s}\right)^2}{2\left(1.60\times10^{-19}\text{C}\right)\left(11.4\times10^3\ \text{N/C}\right)} = \boxed{0.189\,\text{m}}$$

(*b*) Find the elapsed time from constant acceleration relationships. Upon returning to the original position, the final velocity will be the opposite of the initial velocity.

$$v = v_0 + at \ \rightarrow$$

$$t = \frac{v - v_0}{a} = \frac{-2v_0}{\left(-\frac{eE}{m}\right)} = \frac{2mv_0}{eE} = \frac{2\left(9.11\times10^{-31}\text{kg}\right)\left(27.5\times10^6\ \text{m/s}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(11.4\times10^3\ \text{N/C}\right)} = \boxed{2.75\times10^{-8}\text{s}}$$

57. (II) An electron has an initial velocity $\vec{\mathbf{v}}_0 = (8.0 \times 10^4\,\text{m/s})\hat{\mathbf{j}}$. It enters a region where $\vec{\mathbf{E}} = (2.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}) \times 10^4\,\text{N/C}$. (*a*) Determine the vector acceleration of the electron as a function of time. (*b*) At what angle θ is it moving (relative to its initial direction) at $t = 1.0\,\text{ns}$?

57. (*a*) The acceleration is produced by the electric force.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} = -e\vec{\mathbf{E}} \rightarrow$$

$$\vec{\mathbf{a}} = -\frac{e}{m}\vec{\mathbf{E}} = -\frac{(1.60\times10^{-19}\,\text{C})}{(9.11\times10^{-31}\,\text{kg})}\left[(2.0\hat{\mathbf{i}}+8.0\hat{\mathbf{j}})\times10^4\,\text{N/C}\right] = (-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}})\,\text{m/s}^2$$

$$\approx \boxed{-3.5\times10^{15}\,\text{m/s}^2\,\hat{\mathbf{i}} - 1.4\times10^{16}\,\text{m/s}^2\,\hat{\mathbf{j}}}$$

(*b*) The direction is found from the components of the velocity.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t = (8.0\times10^4\,\text{m/s})\hat{\mathbf{j}} + \left[(-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}})\,\text{m/s}^2\right](1.0\times10^{-9}\,\text{s})$$

$$= (-3.513\times10^6\hat{\mathbf{i}} - 1.397\times10^7\hat{\mathbf{j}})\,\text{m/s}$$

$$\tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-1.397\times10^7\,\text{m/s}}{-3.513\times10^6\,\text{m/s}}\right) = 256^\circ \text{ or } -104^\circ$$

This is the direction relative to the x axis. The direction of motion relative to the initial direction is measured from the y axis, and so is $\boxed{\theta = 166^\circ \text{ counter-clockwise}}$ from the initial direction.

63. (II) The HCl molecule has a dipole moment of about $3.4 \times 10^{-30}\,\text{C}\cdot\text{m}$. The two atoms are separated by about $1.0 \times 10^{-10}\,\text{m}$. (*a*) What is the net charge on each atom? (*b*) Is this equal to an integral multiple of e? If not, explain. (*c*) What maximum torque would this dipole experience in a $2.5 \times 10^4\,\text{N/C}$ electric field? (*d*) How much energy would be needed to rotate one molecule 45° from its equilibrium position of lowest potential energy?

63. (*a*) The dipole moment is the effective charge of each atom times the separation distance.

$$p = Q\ell \rightarrow Q = \frac{p}{\ell} = \frac{3.4\times10^{-30}\,\text{C}\bullet\text{m}}{1.0\times10^{-10}\,\text{m}} = \boxed{3.4\times10^{-20}\,\text{C}}$$

(*b*) $\dfrac{Q}{e} = \dfrac{3.4\times10^{-20}\,\text{C}}{1.60\times10^{-19}\,\text{C}} = 0.21$ $\boxed{\text{No}}$, the net charge on each atom is not an integer multiple of e. This is an indication that the H and Cl atoms are not ionized – they haven't fully gained or lost an electron. But rather, the electrons spend more time near the Cl atom than the H atom, giving the molecule a net dipole moment. The electrons are not distributed symmetrically about the two nuclei.

(*c*) The torque is given by Eq. 21-9a.

$$\tau = pE\sin\theta \rightarrow \tau_{\max} = pE = (3.4\times10^{-30}\,\text{C}\bullet\text{m})(2.5\times10^4\,\text{N/C}) = \boxed{8.5\times10^{-26}\,\text{N}\bullet\text{m}}$$

(*d*) The energy needed from an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = (-pE\cos\theta_{\text{final}}) - (-pE\cos\theta_{\text{initial}}) = pE(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}})$$

$$= (3.4\times10^{-30}\,\text{C}\bullet\text{m})(2.5\times10^4\,\text{N/C})[1 - \cos45^\circ] = \boxed{2.5\times10^{-26}\,\text{J}}$$

74. Estimate the net force between the CO group and the HN group shown in Fig. 21–70. The C and O have charges $\pm 0.40e$, and the H and N have charges $\pm 0.20e$, where $e = 1.6 \times 10^{-19}\,\text{C}$. [*Hint*: Do not include the "internal" forces between C and O, or between H and N.]

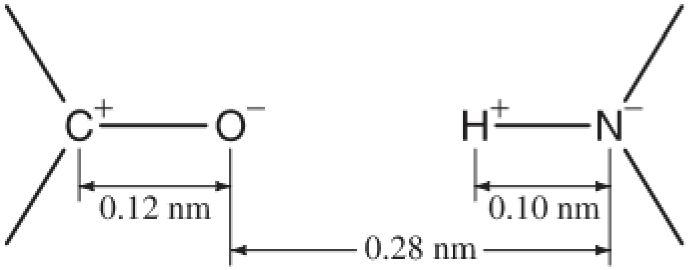


FIGURE 21–70 Problem 74.

85. Suppose electrons enter a uniform electric field midway between two plates at an angle θ_0 to the horizontal, as shown in Fig. 21–75. The path is symmetrical, so they leave at the same angle θ_0 and just barely miss the top plate. What is θ_0? Ignore fringing of the field.

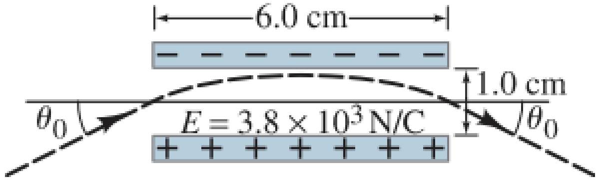


FIGURE 21–75 Problem 85.

74. There are four forces to calculate. Call the rightward direction the positive direction. The value of k is $8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$ and the value of e is $1.602\times10^{-19}\,\text{C}$.

$$F_{\text{net}} = F_{\text{CH}} + F_{\text{CN}} + F_{\text{OH}} + F_{\text{ON}} = \frac{k(0.40e)(0.20e)}{(1\times10^{-9}\,\text{m})^2}\left[-\frac{1}{(0.30)^2}+\frac{1}{(0.40)^2}+\frac{1}{(0.18)^2}-\frac{1}{(0.28)^2}\right]$$

$$= 2.445\times10^{-10}\,\text{N} \approx \boxed{2.4\times10^{-10}\,\text{N}}$$

85. This is a constant acceleration situation, similar to projectile motion in a uniform gravitational field. Let the width of the plates be ℓ, the vertical gap between the plates be h, and the initial velocity be v_0. Notice that the vertical motion has a maximum displacement of $h/2$. Let upwards be the positive vertical direction. We calculate the vertical acceleration produced by the electric field and the time t for the electron to cross the region of the field. We then use constant acceleration equations to solve for the angle.

$$F_y = ma_y = qE = -eE \;\rightarrow\; a_y = -\frac{eE}{m}\;;\; \ell = v_0\cos\theta_0(t) \;\rightarrow\; t = \frac{\ell}{v_0\cos\theta_0}$$

$$v_{y_{\text{top}}} = v_{0y} + a_y t_{\text{top}} \;\rightarrow\; 0 = v_0\sin\theta_0 - \frac{eE}{m}\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) \;\rightarrow\; v_0^2 = \frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)$$

$$y_{\text{top}} = y_0 + v_{0y}t_{\text{top}} + \tfrac{1}{2}a_y t^2 \;\rightarrow\; \tfrac{1}{2}h = v_0\sin\theta_0\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) - \tfrac{1}{2}\frac{eE}{m}\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right)^2 \;\rightarrow$$

$$h = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{v_0^2} = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{\frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)} = \ell\tan\theta_0 - \tfrac{1}{2}\ell\tan\theta_0$$

$$h = \tfrac{1}{2}\ell\tan\theta_0 \;\rightarrow\; \theta_0 = \tan^{-1}\frac{2h}{\ell} = \tan^{-1}\frac{2(1.0\,\text{cm})}{6.0\,\text{cm}} = \boxed{18^\circ}$$

88. A point charge ($m = 1.0\ \mathrm{g}$) at the end of an insulating cord of length 55 cm is observed to be in equilibrium in a uniform horizontal electric field of 15,000 N/C, when the pendulum's position is as shown in Fig. 21–78, with the charge 12 cm above the lowest (vertical) position. If the field points to the right in Fig. 21–78, determine the magnitude and sign of the point charge.

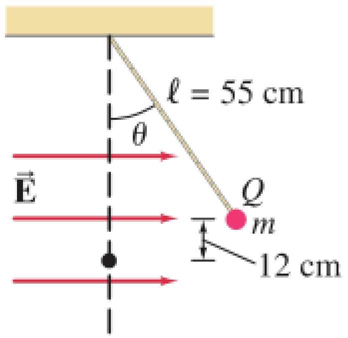
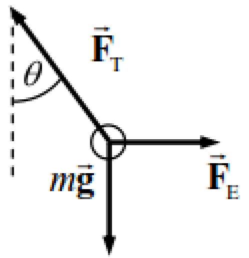
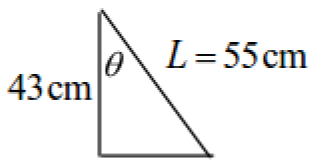


FIGURE 21–78
Problem 88.

88. Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is positive. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions, and use those equations to find the magnitude of the charge.





$$\theta = \cos^{-1}\frac{43\,\mathrm{cm}}{55\,\mathrm{cm}} = 38.6^\circ$$

$$\sum F_x = F_E - F_T \sin\theta = 0 \ \rightarrow\ F_E = F_T \sin\theta = QE$$

$$\sum F_y = F_T \cos\theta - mg = 0 \ \rightarrow\ F_T = \frac{mg}{\cos\theta} \ \rightarrow\ QE = mg\tan\theta$$

$$Q = \frac{mg\tan\theta}{E} = \frac{(1.0\times10^{-3}\,\mathrm{kg})(9.80\,\mathrm{m/s^2})\tan 38.6^\circ}{(1.5\times10^4\,\mathrm{N/C})} = \boxed{5.2\times10^{-7}\,\mathrm{C}}$$