17. Consider the following vector quantities: displacement, velocity, acceleration, momentum, angular momentum, torque. (*a*) Which of these are independent of the choice of origin of coordinates? (Consider different points as origin which are at rest with respect to each other.) (*b*) Which are independent of the velocity of the coordinate system?

17. (*a*) Displacement, velocity, acceleration, and momentum are independent of the choice of origin.
(*b*) Displacement, acceleration, and torque are independent of the velocity of the coordinate system.

7. If all the components of the vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ were reversed in direction, how would this alter $\vec{\mathbf{V}}_1 \times \vec{\mathbf{V}}_2$?

8. Name the four different conditions that could make $\vec{\mathbf{V}}_1 \times \vec{\mathbf{V}}_2 = 0$.

9. A force $\vec{\mathbf{F}} = F\hat{\mathbf{j}}$ is applied to an object at a position $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ where the origin is at the CM. Does the torque about the CM depend on x? On y? On z?

7. The cross product remains the same. $\vec{\mathbf{V}}_1 \times \vec{\mathbf{V}}_2 = (-\vec{\mathbf{V}}_1) \times (-\vec{\mathbf{V}}_2)$

8. The cross product of the two vectors will be zero if the magnitude of either vector is zero or if the vectors are parallel or anti-parallel to each other.

9. The torque about the CM, which is the cross product between **r** and **F**, depends on x and z, but not on y.

3. (II) A person stands, hands at his side, on a platform that is rotating at a rate of 0.90 rev/s. If he raises his arms to a horizontal position, Fig. 11–30, the speed of rotation decreases to 0.70 rev/s. (*a*) Why? (*b*) By what factor has his moment of inertia changed?

FIGURE 11–30 Problem 3.

3. (*a*) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

(*b*) $L_i = L_f \rightarrow I_i\omega_i = I_f\omega_f \rightarrow I_f = I_i\dfrac{\omega_i}{\omega_f} = I_i\dfrac{0.90\,\text{rev/s}}{0.70\,\text{rev/s}} = 1.286\,I_i \approx 1.3\,I_i$

The rotational inertia has increased by a factor of $\boxed{1.3}$.

6. (II) A uniform horizontal rod of mass M and length ℓ rotates with angular velocity ω about a vertical axis through its center. Attached to each end of the rod is a small mass m. Determine the angular momentum of the system about the axis.

6. The angular momentum is the total moment of inertia times the angular velocity.

$$L = I\omega = \left[\tfrac{1}{12}M\ell^2 + 2m\left(\tfrac{1}{2}\ell\right)^2\right]\omega = \boxed{\left(\tfrac{1}{12}M + \tfrac{1}{2}m\right)\ell^2\omega}$$

15. (II) A nonrotating cylindrical disk of moment of inertia I is dropped onto an identical disk rotating at angular speed ω. Assuming no external torques, what is the final common angular speed of the two disks?

15. Since there are no external torques on the system, the angular momentum of the 2-disk system is conserved. The two disks have the same final angular velocity.

$$L_i = L_f \rightarrow I\omega + I(0) = 2I\omega_f \rightarrow \boxed{\omega_f = \tfrac{1}{2}\omega}$$

20. (I) If vector $\vec{\mathbf{A}}$ points along the negative x axis and vector $\vec{\mathbf{B}}$ along the positive z axis, what is the direction of (a) $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ and (b) $\vec{\mathbf{B}} \times \vec{\mathbf{A}}$? ($c$) What is the magnitude of $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ and $\vec{\mathbf{B}} \times \vec{\mathbf{A}}$?

21. (I) Show that (a) $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$, ($b$) $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$, and $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$.

20. We use the determinant rule, Eq. 11-3b.

(a) $$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -A & 0 & 0 \\ 0 & 0 & B \end{vmatrix} = \hat{\mathbf{i}}[(0)(B)-(0)(0)] + \hat{\mathbf{j}}[(0)(0)-(-A)(B)] + \hat{\mathbf{k}}[(-A)(0)-(0)(0)]$$

$$= AB\hat{\mathbf{j}}$$

So the direction of $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ is in the $\boxed{\hat{\mathbf{j}}}$ direction.

(b) Based on Eq. 11-4b, we see that interchanging the two vectors in a cross product reverses the direction. So the direction of $\vec{\mathbf{B}} \times \vec{\mathbf{A}}$ is in the $\boxed{-\hat{\mathbf{j}}}$ direction.

(c) Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular, we have $\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = \left|\vec{\mathbf{B}} \times \vec{\mathbf{A}}\right| = AB\sin 90° = \boxed{AB}$.

21. (a) For all three expressions, use the fact that $\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = AB\sin\theta$. If both vectors in the cross product point in the same direction, then the angle between them is $\theta = 0°$. Since $\sin 0° = 0$, a vector crossed into itself will always give 0. Thus $\boxed{\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0}$.

(b) We use the determinant rule (Eq. 11-3b) to evaluate the other expressions.

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{\mathbf{i}}[(0)(0)-(0)(1)] + \hat{\mathbf{j}}[(0)(0)-(1)(0)] + \hat{\mathbf{k}}[(1)(1)-(0)(0)] = \hat{\mathbf{k}}$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{\mathbf{i}}[(0)(1)-(0)(0)] + \hat{\mathbf{j}}[(0)(0)-(1)(1)] + \hat{\mathbf{k}}[(1)(0)-(0)(0)] = -\hat{\mathbf{j}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{\mathbf{i}}[(1)(1)-(0)(0)] + \hat{\mathbf{j}}[(0)(0)-(0)(1)] + \hat{\mathbf{k}}[(0)(0)-(0)(1)] = \hat{\mathbf{i}}$$

22. (I) The directions of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given below for several cases. For each case, state the direction of $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$. (*a*) $\vec{\mathbf{A}}$ points east, $\vec{\mathbf{B}}$ points south. (*b*) $\vec{\mathbf{A}}$ points east, $\vec{\mathbf{B}}$ points straight down. (*c*) $\vec{\mathbf{A}}$ points straight up, $\vec{\mathbf{B}}$ points north. (*d*) $\vec{\mathbf{A}}$ points straight up, $\vec{\mathbf{B}}$ points straight down.

23. (II) What is the angle θ between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, if $|\vec{\mathbf{A}} \times \vec{\mathbf{B}}| = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$?

24. (II) A particle is located at $\vec{\mathbf{r}} = (4.0\hat{\mathbf{i}} + 3.5\hat{\mathbf{j}} + 6.0\hat{\mathbf{k}})$ m. A force $\vec{\mathbf{F}} = (9.0\hat{\mathbf{j}} - 4.0\hat{\mathbf{k}})$ N acts on it. What is the torque, calculated about the origin?

22. (*a*) East cross south is $\boxed{\text{into the ground}}$.
(*b*) East cross straight down is $\boxed{\text{north}}$.
(*c*) Straight up cross north is $\boxed{\text{west}}$.
(*d*) Straight up cross straight down is $\boxed{0}$ (the vectors are anti-parallel).

23. Use the definitions of cross product and dot product, in terms of the angle between the two vectors.

$$\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = \vec{\mathbf{A}} \bullet \vec{\mathbf{B}} \quad \rightarrow \quad AB\left|\sin\theta\right| = AB\cos\theta \quad \rightarrow \quad \left|\sin\theta\right| = \cos\theta$$

This is true only for angles with positive cosines, and so the angle must be in the first or fourth quadrant. Thus the solutions are $\theta = 45°, 315°$. But the angle between two vectors is always taken to be the smallest angle possible, and so $\theta = \boxed{45°}$.

24. We use the determinant rule, Eq. 11-3b, to evaluate the torque.

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4.0 & 3.5 & 6.0 \\ 0 & 9.0 & -4.0 \end{vmatrix} \text{m}\bullet\text{N}$$

$$= \left\{\hat{\mathbf{i}}\left[(3.5)(-4.0) - (6)(9)\right] + \hat{\mathbf{j}}\left[(6)(0) - (-4)(4)\right] + \hat{\mathbf{k}}\left[(4)(9) - (3)(5)\right]\right\}\text{m}\bullet\text{N}$$

$$= \left(-68\hat{\mathbf{i}} + 16\hat{\mathbf{j}} + 36\hat{\mathbf{k}}\right)\text{m}\bullet\text{N}$$

25. (II) Consider a particle of a rigid object rotating about a fixed axis. Show that the tangential and radial vector components of the linear acceleration are:

$$\vec{\mathbf{a}}_{\tan} = \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}} \quad \text{and} \quad \vec{\mathbf{a}}_R = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{v}}.$$

26. (II) (*a*) Show that the cross product of two vectors, $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$, and $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$ is

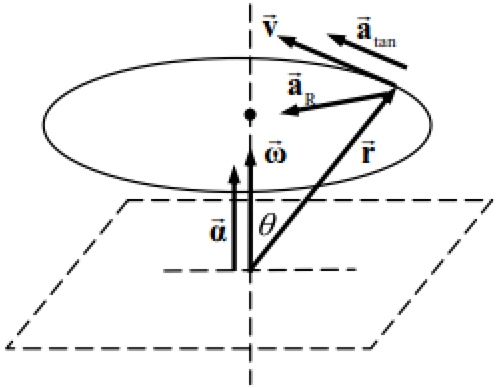
$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} + (A_zB_x - A_xB_z)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}.$$

(*b*) Then show that the cross product can be written

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix},$$

where we use the rules for evaluating a determinant. (Note, however, that this is not really a determinant, but a memory aid.)

25. We choose coordinates so that the plane in which the particle rotates is the *x-y* plane, and so the angular velocity is in the *z* direction. The object is rotating in a circle of radius $r\sin\theta$, where θ is the angle between the position vector and the axis of rotation. Since the object is rigid and rotates about a fixed axis, the linear and angular velocities of the particle are related by $v = \omega r\sin\theta$. The magnitude of the tangential acceleration is $a_{\tan} = \alpha r\sin\theta$. The radial acceleration is given by



$a_R = \dfrac{v^2}{r\sin\theta} = v\dfrac{v}{r\sin\theta} = v\omega$. We assume the object is gaining speed. See the diagram showing the various vectors involved.

The velocity and tangential acceleration are parallel to each other, and the angular velocity and angular acceleration are parallel to each other. The radial acceleration is perpendicular to the velocity, and the velocity is perpendicular to the angular velocity.

We see from the diagram that, using the right hand rule, the direction of $\vec{\mathbf{a}}_R$ is in the direction of $\vec{\boldsymbol{\omega}}\times\vec{\mathbf{v}}$. Also, since $\vec{\boldsymbol{\omega}}$ and $\vec{\mathbf{v}}$ are perpendicular, we have $|\vec{\boldsymbol{\omega}}\times\vec{\mathbf{v}}| = \omega v$ which from above is $v\omega = a_R$. Since both the magnitude and direction check out, we have $\boxed{\vec{\mathbf{a}}_R = \vec{\boldsymbol{\omega}}\times\vec{\mathbf{v}}}$.

We also see from the diagram that, using the right hand rule, the direction of $\vec{\mathbf{a}}_{\tan}$ is in the direction of $\vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}$. The magnitude of $\vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}$ is $|\vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}| = \alpha r\sin\theta$, which from above is $\alpha r\sin\theta = a_{\tan}$. Since both the magnitude and direction check out, we have $\boxed{\vec{\mathbf{a}}_{\tan} = \vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}}$.

26. (II) (*a*) Show that the cross product of two vectors, $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$, and $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$ is

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} + (A_zB_x - A_xB_z)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}.$$

(*b*) Then show that the cross product can be written

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix},$$

where we use the rules for evaluating a determinant. (Note, however, that this is not really a determinant, but a memory aid.)

26. (*a*) We use the distributive property, Eq. 11-4c, to obtain 9 single-term cross products.

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) \times \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$
$$= A_xB_x\left(\hat{\mathbf{i}}\times\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\times\hat{\mathbf{k}}\right) + A_yB_x\left(\hat{\mathbf{j}}\times\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\times\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\times\hat{\mathbf{k}}\right)$$
$$+ A_zB_x\left(\hat{\mathbf{k}}\times\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\times\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\times\hat{\mathbf{k}}\right)$$

Each of these cross products of unit vectors is evaluated using the results of Problem 21 and Eq. 11-4b.

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = A_xB_x(0) + A_xB_y\hat{\mathbf{k}} + A_xB_z\left(-\hat{\mathbf{j}}\right) + A_yB_x\left(-\hat{\mathbf{k}}\right) + A_yB_y(0) + A_yB_z\hat{\mathbf{i}}$$
$$+ A_zB_x\hat{\mathbf{j}} + A_zB_y\left(-\hat{\mathbf{i}}\right) + A_zB_z(0)$$
$$= A_xB_y\hat{\mathbf{k}} - A_xB_z\hat{\mathbf{j}} - A_yB_x\hat{\mathbf{k}} + A_yB_z\hat{\mathbf{i}} + A_zB_x\hat{\mathbf{j}} - A_zB_y\hat{\mathbf{i}}$$

$$= \boxed{\left(A_yB_z - A_zB_y\right)\hat{\mathbf{i}} + \left(A_zB_x - A_xB_z\right)\hat{\mathbf{j}} + \left(A_xB_y - A_yB_x\right)\hat{\mathbf{k}}}$$

(*b*) The rules for evaluating a literal determinant of a 3 x 3 matrix are as follows. The indices on the matrix elements identify the row and column of the element, respectively.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\left(a_{22}a_{33} - a_{23}a_{32}\right) + a_{12}\left(a_{23}a_{31} - a_{21}a_{33}\right) + a_{13}\left(a_{21}a_{32} - a_{22}a_{31}\right)$$

Apply this as a pattern for finding the cross product of two vectors.

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \boxed{\hat{\mathbf{i}}\left(A_yB_z - A_zB_y\right) + \hat{\mathbf{j}}\left(A_zB_x - A_xB_z\right) + \hat{\mathbf{k}}\left(A_xB_y - A_yB_x\right)}$$

This is the same expression as found in part (*a*).

32. (I) What are the x, y, and z components of the angular momentum of a particle located at $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ which has momentum $\vec{\mathbf{p}} = p_x\hat{\mathbf{i}} + p_y\hat{\mathbf{j}} + p_z\hat{\mathbf{k}}$?

32. We use the determinant rule, Eq. 11-3b, to evaluate the angular momentum.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix} = \boxed{(yp_z - zp_y)\hat{\mathbf{i}} + (zp_x - xp_z)\hat{\mathbf{j}} + (xp_y - yp_x)\hat{\mathbf{k}}}$$

34. (I) Calculate the angular momentum of a particle of mass m moving with constant velocity v for two cases (see Fig. 11–33): (*a*) about origin O, and (*b*) about O′.

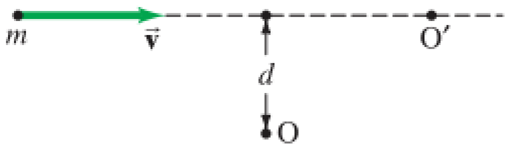


FIGURE 11–33
Problem 34.

35. (II) Two identical particles have equal but opposite momenta, $\vec{\mathbf{p}}$ and $-\vec{\mathbf{p}}$, but they are not traveling along the same line. Show that the total angular momentum of this system does not depend on the choice of origin.

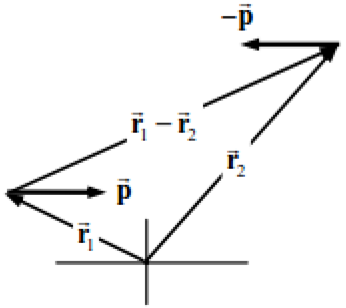
34. (*a*) See Figure 11-33 in the textbook. We have that $L = r_\perp p = dmv$. The direction is into the plane of the page.

(*b*) Since the velocity (and momentum) vectors pass through O′, $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$ are parallel, and so $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = 0$. Or, $r_\perp = 0$, and so $L = 0$.

35. See the diagram. Calculate the total angular momentum about the origin.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_1 \times \vec{\mathbf{p}} + \vec{\mathbf{r}}_2 \times (-\vec{\mathbf{p}}) = (\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \times \vec{\mathbf{p}}$$

The position dependence of the total angular momentum only depends on the difference in the two position vectors. That difference is the same no matter where the origin is chosen, because it is the relative distance between the two particles.



39. (II) Four identical particles of mass m are mounted at equal intervals on a thin rod of length ℓ and mass M, with one mass at each end of the rod. If the system is rotated with angular velocity ω about an axis perpendicular to the rod through one of the end masses, determine (*a*) the kinetic energy and (*b*) the angular momentum of the system.

39. The rotational inertia of the compound object is the sum of the individual moments of inertia.

$$I = I_{\text{particles}} + I_{\text{rod}} = m(0)^2 + m\left(\tfrac{1}{3}\ell\right)^2 + m\left(\tfrac{2}{3}\ell\right)^2 + m\ell^2 + \tfrac{1}{3}M\ell^2 = \left(\tfrac{14}{9}m + \tfrac{1}{3}M\right)\ell^2$$

(*a*) $K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{14}{9}m + \tfrac{1}{3}M\right)\ell^2\omega^2 = \boxed{\left(\tfrac{7}{9}m + \tfrac{1}{6}M\right)\ell^2\omega^2}$

(*b*) $L = I\omega = \boxed{\left(\tfrac{14}{9}m + \tfrac{1}{3}M\right)\ell^2\omega}$

48. (II) A uniform stick 1.0 m long with a total mass of 270 g is pivoted at its center. A 3.0-g bullet is shot through the stick midway between the pivot and one end (Fig. 11–36). The bullet approaches at 250 m/s and leaves at 140 m/s. With what angular speed is the stick spinning after the collision?

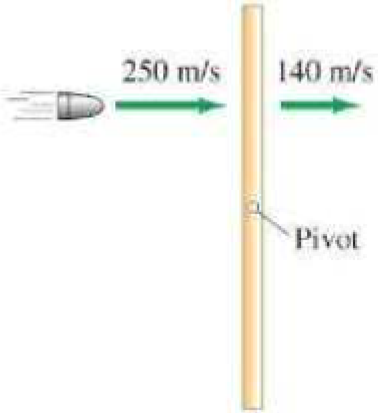


FIGURE 11–36
Problems 48 and 83.

48. Angular momentum about the pivot is conserved during this collision. Note that both objects have angular momentum after the collision.

$$L_{\substack{\text{before}\\\text{collision}}} = L_{\substack{\text{after}\\\text{collision}}} \rightarrow L_{\substack{\text{bullet}\\\text{initial}}} = L_{\substack{\text{stick}\\\text{final}}} + L_{\substack{\text{bullet}\\\text{final}}} \rightarrow m_{\text{bullet}}v_0\left(\tfrac{1}{4}\ell\right) = I_{\text{stick}}\omega + m_{\text{bullet}}v_f\left(\tfrac{1}{4}\ell\right) \rightarrow$$

$$\omega = \frac{m_{\text{bullet}}(v_0 - v_f)\left(\tfrac{1}{4}\ell\right)}{I_{\text{stick}}} = \frac{m_{\text{bullet}}(v_0 - v_f)\left(\tfrac{1}{4}\ell\right)}{\tfrac{1}{12}M_{\text{stick}}\ell_{\text{stick}}^2} = \frac{3m_{\text{bullet}}(v_0 - v_f)}{M_{\text{stick}}\ell_{\text{stick}}} = \frac{3(0.0030\,\text{kg})(110\,\text{m/s})}{(0.27\,\text{kg})(1.0\,\text{m})}$$

$$= \boxed{3.7\,\text{rad/s}}$$

65. A particle of mass 1.00 kg is moving with velocity $\vec{\mathbf{v}} = (7.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}})$ m/s. (*a*) Find the angular momentum $\vec{\mathbf{L}}$ relative to the origin when the particle is at $\vec{\mathbf{r}} = (2.0\hat{\mathbf{j}} + 4.0\hat{\mathbf{k}})$ m. (*b*) At position $\vec{\mathbf{r}}$ a force of $\vec{\mathbf{F}} = 4.0\,\text{N}\,\hat{\mathbf{i}}$ is applied to the particle. Find the torque relative to the origin.

65. (*a*) Use Eq. 11-6 to find the angular momentum.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = m(\vec{\mathbf{r}} \times \vec{\mathbf{v}}) = (1.00\,\text{kg})\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2.0 & 4.0 \\ 7.0 & 6.0 & 0 \end{vmatrix}\text{kg}\cdot\text{m}^2/\text{s} = \boxed{\left(-24\hat{\mathbf{i}} + 28\hat{\mathbf{j}} - 14\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m}^2/\text{s}}$$

(*b*) $$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2.0 & 4.0 \\ 4.0 & 0 & 0 \end{vmatrix}\text{m}\cdot\text{N} = \boxed{\left(16\hat{\mathbf{j}} - 8.0\hat{\mathbf{k}}\right)\text{m}\cdot\text{N}}$$

70. The position of a particle with mass m traveling on a helical path (see Fig. 11–45) is given by

$$\vec{\mathbf{r}} = R\cos\left(\frac{2\pi z}{d}\right)\hat{\mathbf{i}} + R\sin\left(\frac{2\pi z}{d}\right)\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

where R and d are the radius and pitch of the helix, respectively, and z has time dependence $z = v_z t$ where v_z is the (constant) component of velocity in the z direction. Determine the time-dependent angular momentum $\vec{\mathbf{L}}$ of the particle about the origin.

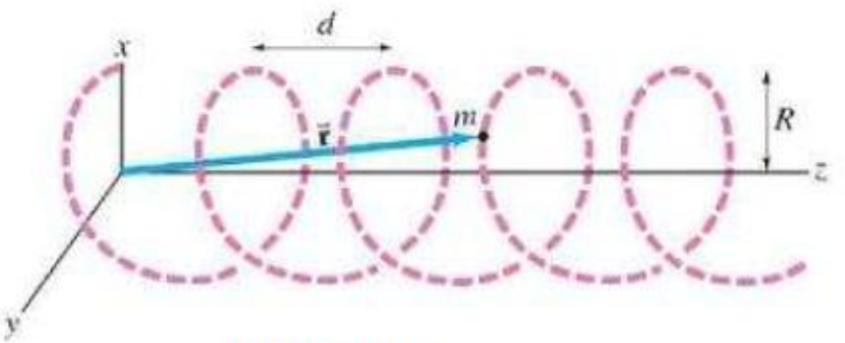


FIGURE 11–45 Problem 70.

70. Note that $z = v_z t$, and so $\dfrac{dz}{dt} = v_z$. To find the angular momentum, use Eq. 11-6, $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$.

$$\vec{\mathbf{r}} = R\cos\left(\frac{2\pi z}{d}\right)\hat{\mathbf{i}} + R\sin\left(\frac{2\pi z}{d}\right)\hat{\mathbf{j}} + z\hat{\mathbf{k}} = R\cos\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{i}} + R\sin\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{j}} + v_z t\hat{\mathbf{k}}$$

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = -R\frac{2\pi v_z}{d}\sin\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{i}} + R\frac{2\pi v_z}{d}\cos\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

To simplify the notation, let $\alpha \equiv \dfrac{2\pi v_z}{d}$. Then the kinematical expressions are as follows.

$$\vec{\mathbf{r}} = R\cos(\alpha t)\hat{\mathbf{i}} + R\sin(\alpha t)\hat{\mathbf{j}} + v_z t\hat{\mathbf{k}}\ ;\ \vec{\mathbf{v}} = -\alpha R\sin(\alpha t)\hat{\mathbf{i}} + \alpha R\cos(\alpha)\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = m\vec{\mathbf{r}} \times \vec{\mathbf{v}} = m\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ R\cos(\alpha t) & R\sin(\alpha t) & v_z t \\ -R\alpha\sin(\alpha t) & R\alpha\cos(\alpha t) & v_z \end{vmatrix}$$

$$= m\left[Rv_z\sin(\alpha t) - R\alpha v_z t\cos(\alpha t)\right]\hat{\mathbf{i}} + m\left[-R\alpha v_z t\sin(\alpha t) - Rv_z\cos(\alpha t)\right]\hat{\mathbf{j}}$$
$$+ m\left[R^2\alpha\cos^2(\alpha t) + R^2\alpha\sin^2(\alpha t)\right]\hat{\mathbf{k}}$$

$$= mRv_z\left[\sin(\alpha t) - \alpha t\cos(\alpha t)\right]\hat{\mathbf{i}} + mRv_z\left[-\alpha t\sin(\alpha t) - \cos(\alpha t)\right]\hat{\mathbf{j}} + mR^2\alpha\hat{\mathbf{k}}$$

$$= mRv_z\left\{\left[\sin(\alpha t) - \alpha t\cos(\alpha t)\right]\hat{\mathbf{i}} + \left[-\alpha t\sin(\alpha t) - \cos(\alpha t)\right]\hat{\mathbf{j}} + \frac{R\alpha}{v_z}\hat{\mathbf{k}}\right\}$$

$$= \boxed{mRv_z\left\{\left[\sin\left(\frac{2\pi z}{d}\right) - \frac{2\pi z}{d}\cos\left(\frac{2\pi z}{d}\right)\right]\hat{\mathbf{i}} + \left[-\frac{2\pi z}{d}\sin\left(\frac{2\pi z}{d}\right) - \cos\left(\frac{2\pi z}{d}\right)\right]\hat{\mathbf{j}} + \frac{2\pi R}{d}\mathbf{k}\right\}}$$