2. Suppose a disk rotates at constant angular velocity. Does a point on the rim have radial and/or tangential acceleration? If the disk's angular velocity increases uniformly, does the point have radial and/or tangential acceleration? For which cases would the magnitude of either component of linear acceleration change?

3. Could a nonrigid object be described by a single value of the angular velocity ω? Explain.

2. A point on the rim of a disk rotating with constant angular velocity has no tangential acceleration since the tangential speed is constant. It does have radial acceleration. Although the point's speed is not changing, its velocity is, since the velocity vector is changing direction. The point has a centripetal acceleration, which is directed radially inward. If the disk's angular velocity increases uniformly, the point on the rim will have both radial and tangential acceleration, since it is both moving in a circle and speeding up. The magnitude of the radial component of acceleration will increase in the case of the disk with a uniformly increasing angular velocity, although the tangential component will be constant. In the case of the disk rotating with constant angular velocity, neither component of linear acceleration will change.

3. No. The relationship between the parts of a non-rigid object can change. Different parts of the object may have different values of ω.

9. Two spheres look identical and have the same mass. However, one is hollow and the other is solid. Describe an experiment to determine which is which.

9. Roll the spheres down an incline. The hollow sphere will have a great moment of inertia and will take longer to reach the bottom of the incline.

14. The moment of inertia of a rotating solid disk about an axis through its CM is $\frac{1}{2}MR^2$ (Fig. 10–20c). Suppose instead that a parallel axis of rotation passes through a point on the edge of the disk. Will the moment of inertia be the same, larger, or smaller?

14. Larger. The moment of inertia depends on the distribution of mass. Imagine the disk as a collection of many little bits of mass. Moving the axis of rotation to the edge of the disk increases the average distance of the bits of mass to the axis, and therefore increases the moment of inertia. (See the Parallel Axis theorem.)

1. (I) Express the following angles in radians: (*a*) 45.0°, (*b*) 60.0°, (*c*) 90.0°, (*d*) 360.0°, and (*e*) 445°. Give as numerical values and as fractions of π.

1. (*a*) $(45.0°)(2\pi\text{ rad}/360°) = \boxed{\pi/4\text{ rad}} = \boxed{0.785\text{ rad}}$

(*b*) $(60.0°)(2\pi\text{ rad}/360°) = \boxed{\pi/3\text{ rad}} = \boxed{1.05\text{ rad}}$

(*c*) $(90.0°)(2\pi\text{ rad}/360°) = \boxed{\pi/2\text{ rad}} = \boxed{1.57\text{ rad}}$

(*d*) $(360.0°)(2\pi\text{ rad}/360°) = \boxed{2\pi\text{ rad}} = \boxed{6.283\text{ rad}}$

(*e*) $(445°)(2\pi\text{ rad}/360°) = \boxed{89\pi/36\text{ rad}} = \boxed{7.77\text{ rad}}$

14. (II) A turntable of radius R_1 is turned by a circular rubber roller of radius R_2 in contact with it at their outer edges. What is the ratio of their angular velocities, ω_1/ω_2?

14. The tangential speed of the turntable must be equal to the tangential speed of the roller, if there is no slippage.

$$v_1 = v_2 \rightarrow \omega_1 R_1 = \omega_2 R_2 \rightarrow \boxed{\omega_1/\omega_2 = R_2/R_1}$$

17. (I) A centrifuge accelerates uniformly from rest to 15,000 rpm in 220 s. Through how many revolutions did it turn in this time?

17. The angular displacement can be found from Eq. 10-9d.

$$\theta = \bar{\omega}t = \tfrac{1}{2}(\omega_o + \omega)t = \tfrac{1}{2}(0 + 15000\,\text{rev/min})(220\,\text{s})(1\,\text{min}/60\,\text{s}) = \boxed{2.8\times10^4\,\text{rev}}$$

22. (II) The angle through which a rotating wheel has turned in time t is given by $\theta = 8.5\,t - 15.0\,t^2 + 1.6\,t^4$, where θ is in radians and t in seconds. Determine an expression (*a*) for the instantaneous angular velocity ω and (*b*) for the instantaneous angular acceleration α. (*c*) Evaluate ω and α at $t = 3.0\,\text{s}$. (*d*) What is the average angular velocity, and (*e*) the average angular acceleration between $t = 2.0\,\text{s}$ and $t = 3.0\,\text{s}$?

22. We are given that $\theta = 8.5t - 15.0t^2 + 1.6t^4$.

(*a*) $\omega = \frac{d\theta}{dt} = \boxed{8.5 - 30.0t + 6.4t^3}$, where ω is in rad/sec and t is in sec.

(*b*) $\alpha = \frac{d\omega}{dt} = \boxed{-30.0 + 19.2t^2}$, where α is in rad/sec^2 and t is in sec.

(*c*) $\omega(3.0) = 8.5 - 30.0(3.0) + 6.4(3.0)^3 = \boxed{91\,\text{rad/s}}$

$\alpha(3.0) = -30.0 + 19.2(3.0)^2 = \boxed{140\,\text{rad/s}^2}$

(*d*) The average angular velocity is the angular displacement divided by the elapsed time.

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta(3.0) - \theta(2.0)}{3.0\,\text{s} - 2.0\,\text{s}}$$

$$= \frac{\left[8.5(3.0) - 15.0(3.0)^2 + 1.6(3.0)^4\right] - \left[8.5(2.0) - 15.0(2.0)^2 + 1.6(2.0)^4\right]}{1.0\,\text{s}}$$

$$= \boxed{38\,\text{rad/s}}$$

28. (II) A wheel of diameter 27.0 cm is constrained to rotate in the xy plane, about the z axis, which passes through its center. A force $\vec{\mathbf{F}} = (-31.0\hat{\mathbf{i}} + 43.4\hat{\mathbf{j}})$ N acts at a point on the edge of the wheel that lies exactly on the x axis at a particular instant. What is the torque about the rotation axis at this instant?

28. The lever arm to the point of application of the force is along the x axis. Thus the perpendicular part of the force is the y component. Use Eq. 10-10b.

$$\tau = RF_{\perp} = (0.135\,\text{m})(43.4\,\text{N}) = \boxed{5.86\,\text{m}\cdot\text{N, counterclockwise}}$$

30. (II) Determine the net torque on the 2.0-m-long uniform beam shown in Fig. 10–50. Calculate about (a) point C, the CM, and (b) point P at one end.

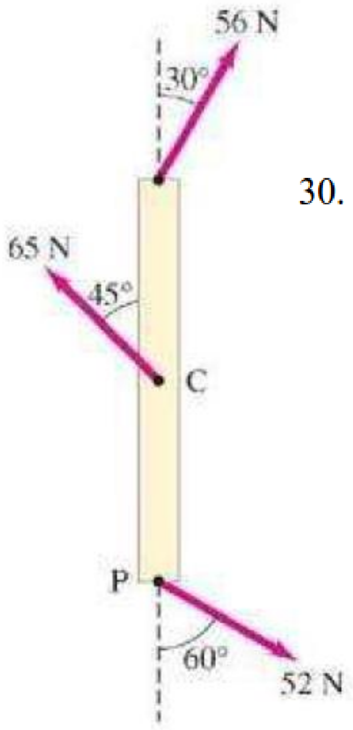


FIGURE 10–50
Problem 30.

30. For each torque, use Eq. 10-10c. Take counterclockwise torques to be positive.

(a) Each force has a lever arm of 1.0 m.

$$\tau_{\substack{\text{about}\\ \text{C}}} = -(1.0\,\text{m})(56\,\text{N})\sin 30^\circ + (1.0\,\text{m})(52\,\text{N})\sin 60^\circ = \boxed{17\,\text{m}\cdot\text{N}}$$

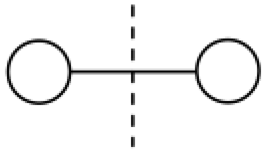
(b) The force at C has a lever arm of 1.0 m, and the force at the top has a lever arm of 2.0 m.

$$\tau_{\substack{\text{about}\\ \text{P}}} = -(2.0\,\text{m})(56\,\text{N})\sin 30^\circ + (1.0\,\text{m})(65\,\text{N})\sin 45^\circ = \boxed{-10\,\text{m}\cdot\text{N}} \text{ (2 sig fig)}$$

The negative sign indicates a clockwise torque.

34. (II) An oxygen molecule consists of two oxygen atoms whose total mass is 5.3×10^{-26} kg and whose moment of inertia about an axis perpendicular to the line joining the two atoms, midway between them, is $1.9 \times 10^{-46}\ \mathrm{kg \cdot m^2}$. From these data, estimate the effective distance between the atoms.

34. The oxygen molecule has a "dumbbell" geometry, rotating about the dashed line, as shown in the diagram. If the total mass is M, then each atom has a mass of $M/2$. If the distance between them is d, then the distance from the axis of rotation to each atom is $d/2$. Treat each atom as a particle for calculating the moment of inertia.

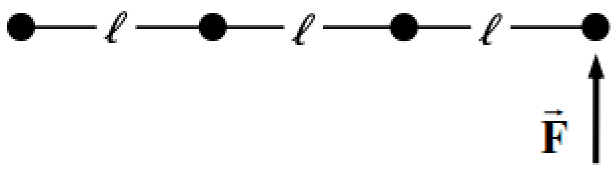


$$I = (M/2)(d/2)^2 + (M/2)(d/2)^2 = 2(M/2)(d/2)^2 = \tfrac{1}{4}Md^2 \quad \rightarrow$$

$$d = \sqrt{4I/M} = \sqrt{4\left(1.9\times10^{-46}\,\mathrm{kg \bullet m^2}\right)/\left(5.3\times10^{-26}\,\mathrm{kg}\right)} = \boxed{1.2\times10^{-10}\,\mathrm{m}}$$

45. (II) Four equal masses M are spaced at equal intervals, ℓ, along a horizontal straight rod whose mass can be ignored. The system is to be rotated about a vertical axis passing through the mass at the left end of the rod and perpendicular to it. (*a*) What is the moment of inertia of the system about this axis? (*b*) What minimum force, applied to the farthest mass, will impart an angular acceleration α? (*c*) What is the direction of this force?

45. Each mass is treated as a point particle. The first mass is at the axis of rotation; the second mass is a distance ℓ from the axis of rotation; the third mass is 2ℓ from the axis, and the fourth mass is 3ℓ from the axis.



(*a*) $I = M\ell^2 + M(2\ell)^2 + M(3\ell)^2 = \boxed{14M\ell^2}$

(*b*) The torque to rotate the rod is the perpendicular component of force times the lever arm, and is also the moment of inertia times the angular acceleration.

$$\tau = I\alpha = F_{\perp}r \;\rightarrow\; F_{\perp} = \frac{I\alpha}{r} = \frac{14M\ell^2\alpha}{3\ell} = \boxed{\tfrac{14}{3}M\ell\alpha}$$

(*c*) The force must be perpendicular to the rod connecting the masses, and perpendicular to the axis of rotation. An appropriate direction is shown in the diagram.

58. (II) A ball of mass M and radius r_1 on the end of a thin massless rod is rotated in a horizontal circle of radius R_0 about an axis of rotation AB, as shown in Fig. 10–58. (*a*) Considering the mass of the ball to be concentrated at its center of mass, calculate its moment of inertia about AB. (*b*) Using the parallel-axis theorem and considering the finite radius of the ball, calculate the moment of inertia of the ball about AB. (*c*) Calculate the percentage error introduced by the point mass approximation for $r_1 = 9.0\text{ cm}$ and $R_0 = 1.0\text{ m}$.

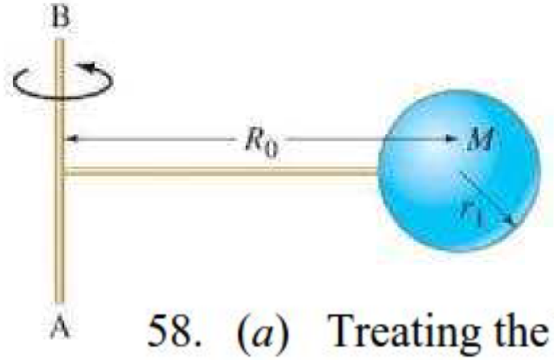


FIGURE 10–58 Problem 58.

58. (*a*) Treating the ball as a point mass, the moment of inertia about AB is $I = \boxed{MR_0^2}$.

(*b*) The parallel axis theorem is given in Eq. 10-17. The distance from the center of mass of the ball to the axis of rotation is $h = R_0$.

$$I = I_{\text{CM}} + Mh^2 = \boxed{\tfrac{2}{5}Mr_1^2 + MR_0^2}$$

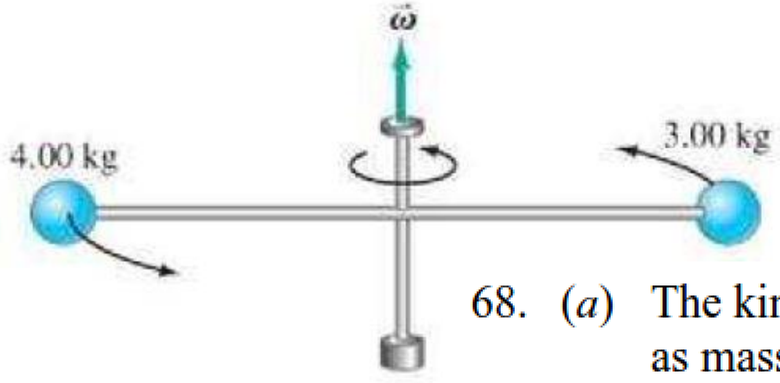
(*c*)

$$\%\text{ error} = \left(\frac{I_{\text{approx}} - I_{\text{exact}}}{I_{\text{exact}}}\right)(100) = \left[\frac{MR_0^2 - \left(\frac{2}{5}Mr_1^2 + MR_0^2\right)}{\frac{2}{5}Mr_1^2 + MR_0^2}\right](100) = \frac{-\frac{2}{5}Mr_1^2}{\frac{2}{5}Mr_1^2 + MR_0^2}(100)$$

$$= \frac{-1}{1 + \frac{5}{2}\left(R_0/r_1\right)^2}(100) = -\frac{-1}{1 + \frac{5}{2}(1.0/0.090)^2}(100) = -0.32295 \approx \boxed{-0.32}$$

The negative sign means that the approximation is smaller than the exact value, by about 0.32%.

68. (III) A 4.00-kg mass and a 3.00-kg mass are attached to opposite ends of a thin 42.0-cm-long horizontal rod (Fig. 10–60). The system is rotating at angular speed $\omega = 5.60\,\mathrm{rad/s}$ about a vertical axle at the center of the rod. Determine (*a*) the kinetic energy K of the system, and (*b*) the net force on each mass. (*c*) Repeat parts (*a*) and (*b*) assuming that the axle passes through the CM of the system.



68. (*a*) The kinetic energy of the system is the kinetic energy of the two masses, since the rod is treated as massless. Let A represent the heavier mass, and B the lighter mass.

$$K = \tfrac{1}{2} I_A \omega_A^2 + \tfrac{1}{2} I_B \omega_B^2 = \tfrac{1}{2} m_A r_A^2 \omega_A^2 + \tfrac{1}{2} m_B r_B^2 \omega_A^2 = \tfrac{1}{2} r^2 \omega^2 (m_A + m_B)$$

$$= \tfrac{1}{2}(0.210\,\mathrm{m})^2 (5.60\,\mathrm{rad/s})^2 (7.00\,\mathrm{kg}) = \boxed{4.84\,\mathrm{J}}$$

(*b*) The net force on each object produces centripetal motion, and so can be expressed as $mr\omega^2$.

$$F_A = m_A r_A \omega_A^2 = (4.00\,\mathrm{kg})(0.210\,\mathrm{m})(5.60\,\mathrm{rad/s})^2 = \boxed{26.3\,\mathrm{N}}$$

$$F_B = m_B r_B \omega_B^2 = (3.00\,\mathrm{kg})(0.210\,\mathrm{m})(5.60\,\mathrm{rad/s})^2 = \boxed{19.8\,\mathrm{N}}$$

These forces are exerted by the rod. Since they are unequal, there would be a net horizontal force on the rod (and hence the axle) due to the masses. This horizontal force would have to be counteracted by the mounting for the rod and axle in order for the rod not to move horizontally. There is also a gravity force on each mass, balanced by a vertical force from the rod, so that there is no net vertical force on either mass.

(*c*) Take the 4.00 kg mass to be the origin of coordinates for determining the center of mass.

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{(4.00\,\text{kg})(0) + (3.00\,\text{kg})(0.420\,\text{m})}{7.00\,\text{kg}} = 0.180\,\text{m from mass A}$$

So the distance from mass A to the axis of rotation is now 0.180 m, and the distance from mass B to the axis of rotation is now 0.24 m. Re-do the above calculations with these values.

$$K = \tfrac{1}{2} I_A \omega_A^2 + \tfrac{1}{2} I_B \omega_B^2 = \tfrac{1}{2} m_A r_A^2 \omega_A^2 + \tfrac{1}{2} m_B r_B^2 \omega_A^2 = \tfrac{1}{2}\omega^2 \left(m_A r_A^2 + m_B r_B^2\right)$$

$$= \tfrac{1}{2}(5.60\,\text{rad/s})^2 \left[(4.00\,\text{kg})(0.180\,\text{m})^2 + (3.00\,\text{kg})(0.240\,\text{m})^2\right] = \boxed{4.74\,\text{J}}$$

$$F_A = m_A r_A \omega_A^2 = (4.00\,\text{kg})(0.180\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{22.6\,\text{N}}$$

$$F_B = m_B r_B \omega_B^2 = (3.00\,\text{kg})(0.240\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{22.6\,\text{N}}$$

Note that the horizontal forces are now equal, and so there will be no horizontal force on the rod or axle.

90. Figure 10–65 illustrates an H_2O molecule. The O—H bond length is 0.96 nm and the H—O—H bonds make an angle of 104°. Calculate the moment of inertia for the H_2O molecule about an axis passing through the center of the oxygen atom (*a*) perpendicular to the plane of the molecule, and (*b*) in the plane of the molecule, bisecting the H—O—H bonds.

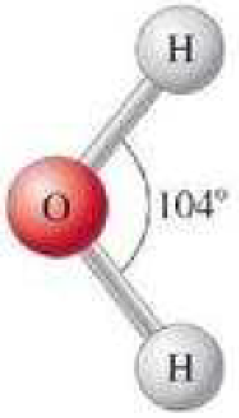
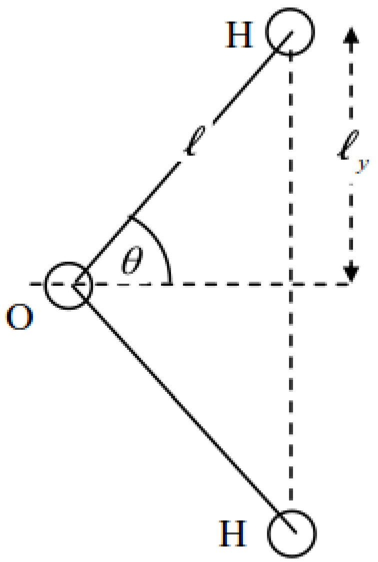


FIGURE 10–65
Problem 90.

90. The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is 1.66×10^{-27} kg. Since the axis passes through the oxygen atom, it will have no rotational inertia.



(*a*) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance ℓ from the axis of rotation.

$$I_{\text{perp}} = 2m_H\ell^2 = 2(1.01)\left(1.66\times10^{-27}\,\text{kg}\right)\left(0.96\times10^{-9}\,\text{m}\right)^2$$

$$= \boxed{3.1\times10^{-45}\,\text{kg}\bullet\text{m}^2}$$

(*b*) If the axis is in the plane of the molecule, bisecting the H-O-H bonds, each hydrogen atom is a distance of $\ell_y = \ell\sin\theta = \left(9.6\times10^{-10}\,\text{m}\right)\sin 52^\circ$ $= 7.564\times10^{-10}\,\text{m}$. Thus the moment of inertia is as follows.

$$I_{\text{plane}} = 2m_H\ell_y^2 = 2(1.01)\left(1.66\times10^{-27}\,\text{kg}\right)\left(7.564\times10^{-10}\,\text{m}\right)^2 = \boxed{1.9\times10^{-45}\,\text{kg}\bullet\text{m}^2}$$

93. A wheel of mass M has radius R. It is standing vertically on the floor, and we want to exert a horizontal force F at its axle so that it will climb a step against which it rests (Fig. 10–66). The step has height h, where $h < R$. What minimum force F is needed?

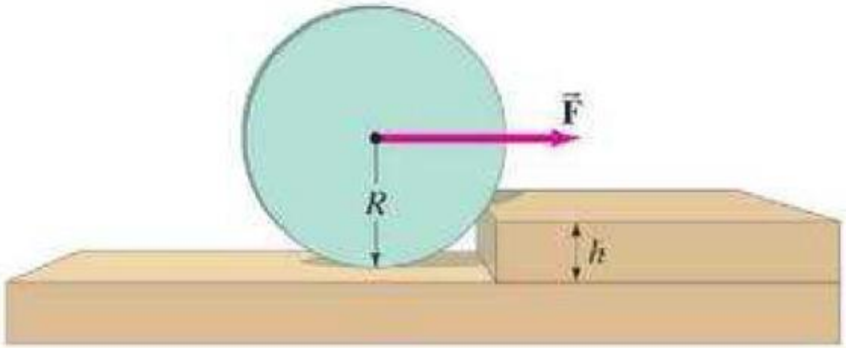
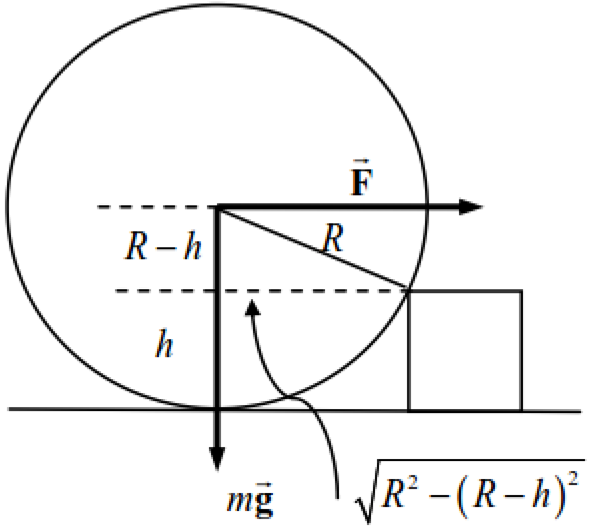


FIGURE 10–6

93. The wheel is rolling about the point of contact with the step, and so all torques are to be taken about that point. As soon as the wheel is off the floor, there will be only two forces that can exert torques on the wheel – the pulling force and the force of gravity. There will not be a normal force of contact between the wheel and the floor once the wheel is off the floor, and any force on the wheel from the point of the step cannot exert a torque about that very point. Calculate the net torque on the wheel, with clockwise torques positive. The minimum force occurs when the net torque is 0.



$$\sum \tau = F(R-h) - mg\sqrt{R^2-(R-h)^2} = 0$$

$$F = \frac{Mg\sqrt{R^2-(R-h)^2}}{R-h} = \boxed{\frac{Mg\sqrt{2Rh-h^2}}{R-h}}$$