1. What would be the appearance of the Moon if it had (*a*) a rough surface; (*b*) a polished mirrorlike surface?

1. (*a*) The Moon would look just like it does now, since the surface is rough. Reflected sunlight is scattered by the surface of the Moon in many directions, making the surface appear white.

   (*b*) With a polished, mirror-like surface, the Moon would reflect an image of the Sun, the stars, and the Earth. The appearance of the Moon would be different as seen from different locations on the Earth.

2. Archimedes is said to have burned the whole Roman fleet in the harbor of Syracuse by focusing the rays of the Sun with a huge spherical mirror. Is this[†] reasonable?

2. Yes, it would have been possible, although certainly difficult. Several attempts have been made to reenact the event in order to test its feasibility. Two of the successful attempts include a 1975 experiment directed by Greek scientist Dr. Ioannis Sakkas and a 2005 experiment performed by a group of engineering students at MIT. (See www.mit.edu for links to both these and other similar experiments.) In both these cases, several individual mirrors operating together simulated a large spherical mirror and were used to ignite a wooden boat. If in fact the story is true, Archimedes would have needed good weather and an enemy fleet that cooperated by staying relatively still while the focused sunlight heated the wood.

17. A ray of light is refracted through three different materials (Fig. 32–43). Rank the materials according to their index of refraction, least to greatest.

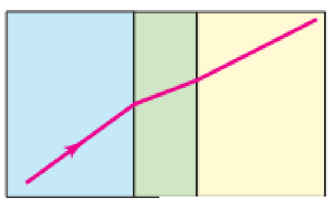


FIGURE 32–43 Question 17.

17. When the light ray passes from the blue material to the green material, the ray bends toward the normal. This indicates that the index of refraction of the blue material is less than that of the green material. When the light ray passes from the green material to the yellow material, the ray bends away from the normal, but not far enough to make the ray parallel to the initial ray, indicating that the index of refraction of the yellow material is less than that of the green material but larger than the index of refraction of the blue material. The ranking of the indices of refraction is, least to greatest, blue, yellow, and green.

3. (II) Two plane mirrors meet at a 135° angle, Fig. 32–45. If light rays strike one mirror at 38° as shown, at what angle ϕ do they leave the second mirror?

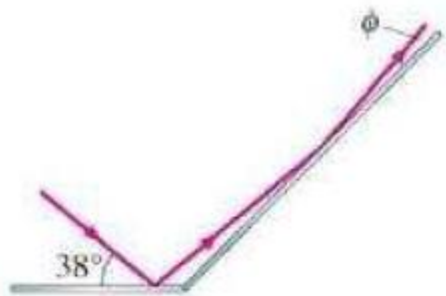
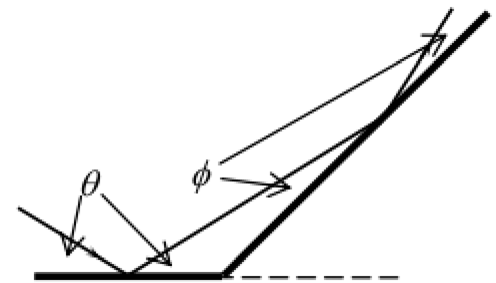


FIGURE 32–45
Problem 3.

3. The law of reflection can be applied twice. At the first reflection, the angle is θ, and at the second reflection, the angle is ϕ. Consider the triangle formed by the mirrors and the first reflected ray.

$$\theta + \alpha + \phi = 180° \rightarrow 38° + 135° + \phi = 180° \rightarrow \boxed{\phi = 7°}$$



5. (II) Show that if two plane mirrors meet at an angle ϕ, a single ray reflected successively from both mirrors is deflected through an angle of 2ϕ independent of the incident angle. Assume $\phi < 90°$ and that only two reflections, one from each mirror, take place.

5. The incoming ray is represented by line segment DA. For the first reflection at A the angles of incidence and reflection are θ_1. For the second reflection at B the angles of incidence and reflection are θ_2. We relate θ_1 and θ_2 to the angle at which the mirrors meet, ϕ, by using the sum of the angles of the triangle ABC.

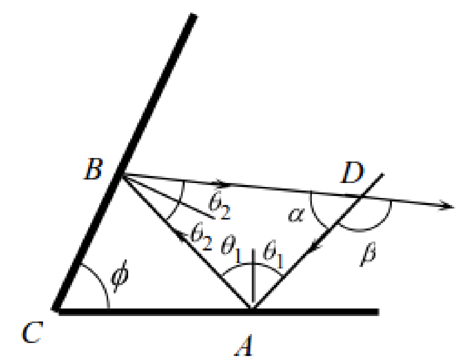
$$\phi + (90° - \theta_1) + (90° - \theta_2) = 180° \rightarrow \phi = \theta_1 + \theta_2$$

Do the same for triangle ABD.

$$\alpha + 2\theta_1 + 2\theta_2 = 180° \rightarrow \alpha = 180° - 2(\theta_1 + \theta_2) = 180° - 2\phi$$

At point D we see that the deflection is as follows.

$$\beta = 180° - \alpha = 180° - (180° - 2\phi) = \boxed{2\phi}$$



17. (II) You are standing 3.0 m from a convex security mirror in a store. You estimate the height of your image to be half of your actual height. Estimate the radius of curvature of the mirror.

17. The object distance of 3.0 m and the magnification of +0.5 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \quad \rightarrow \quad d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{0.5(3.0\,\text{m})}{0.5-1} = -3.0\,\text{m}$$

$$r = 2f = 2(-3.0\,\text{m}) = \boxed{-6.0\,\text{m}}$$

32. (I) The speed of light in ice is 2.29×10^8 m/s. What is the index of refraction of ice?

32. We find the index of refraction from Eq. 32-1.

$$n = \frac{c}{v} = \frac{3.00\times10^8\ \text{m/s}}{2.29\times10^8\ \text{m/s}} = \boxed{1.31}$$

37. (II) Light is emitted from an ordinary lightbulb filament in wave-train bursts of about 10^{-8} s in duration. What is the length in space of such wave trains?

$\boxed{37.}$ The length in space of a burst is the speed of light times the elapsed time.

$$d = ct = (3.00 \times 10^8\ \text{m/s})(10^{-8}\,\text{s}) = \boxed{3\,\text{m}}$$

39. (I) A flashlight beam strikes the surface of a pane of glass ($n = 1.56$) at a 63° angle to the normal. What is the angle of refraction?

39. Find the angle of refraction from Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.56}\sin 63°\right) = \boxed{35°}$$

43. (II) A light beam strikes a 2.0-cm-thick piece of plastic with a refractive index of 1.62 at a 45° angle. The plastic is on top of a 3.0-cm-thick piece of glass for which $n = 1.47$. What is the distance D in Fig. 32–48?

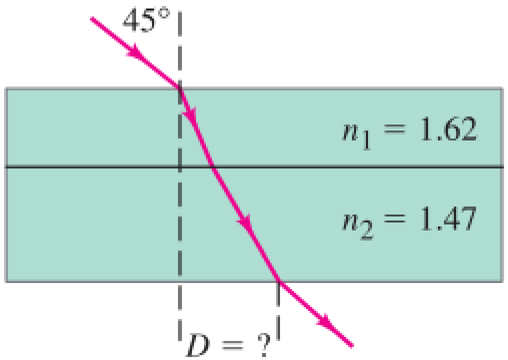
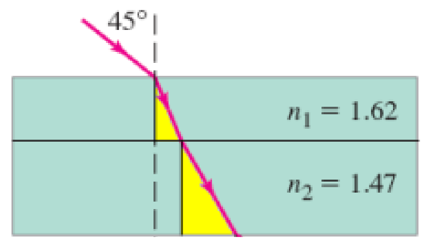


FIGURE 32–48
Problem 43.

43. The beam forms the hypotenuse of two right triangles as it passes through the plastic and then the glass. The upper angle of the triangle is the angle of refraction in that medium. Note that the sum of the opposite sides is equal to the displacement D. First, we calculate the angles of refraction in each medium using Snell's Law (Eq. 32-5).



$$\sin 45 = n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 = \sin^{-1}\left(\frac{\sin 45}{n_1}\right) = \sin^{-1}\left(\frac{\sin 45}{1.62}\right) = 25.88°$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 45}{n_2}\right) = \sin^{-1}\left(\frac{\sin 45}{1.47}\right) = 28.75°$$

We then use the trigonometric identity for tangent to calculate the two opposite sides, and sum to get the displacement.

$$D = D_1 + D_2 = h_1 \tan\theta_1 + h_1 \tan\theta_1 = (2.0\text{cm})\tan 25.88° + (3.0\text{cm})\tan 28.75° = \boxed{2.6\text{cm}}$$

46. (II) The block of glass ($n = 1.5$) shown in cross section in Fig. 32–51 is surrounded by air. A ray of light enters the block at its left-hand face with incident angle θ_1 and reemerges into the air from the right-hand face directed parallel to the block's base. Determine θ_1.

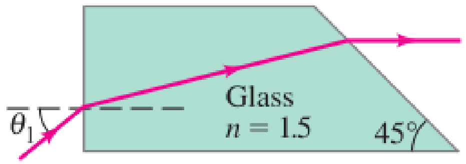
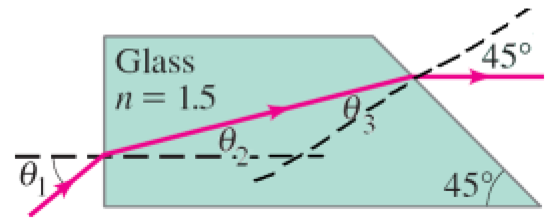


FIGURE 32–51 Problem 46.

46. Since the light ray travels parallel to the base when it exits the glass, and the back edge of the glass makes a 45° angle to the horizontal, the exiting angle of refraction is 45°. We use Snell's law, Eq. 32-5, to calculate the incident angle at the back pane.



$$\theta_3 = \sin^{-1}\left[\frac{n_4}{n_3}\sin\theta_4\right] = \sin^{-1}\left[\frac{1.0}{1.5}\sin\ 45°\right] = 28.13°$$

We calculate the refracted angle at the front edge of the glass by noting that the angles θ_2 and θ_3 in the figure form two angles of a triangle. The third angle, as determined by the perpendiculars to the surface, is 135°.

$$\theta_2 + \theta_3 + 135° = 180° \rightarrow \theta_2 = 45° - \theta_3 = 45° - 28.13° = 16.87°$$

Finally, we use Snell's law at the front face of the glass to calculate the incident angle.

$$\theta_1 = \sin^{-1}\left[\frac{n_2}{n_1}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.5}{1.0}\sin\ 16.87°\right] = 25.81° \approx \boxed{26°}$$

54. (II) A parallel beam of light containing two wavelengths, $\lambda_1 = 465$ nm and $\lambda_2 = 652$ nm, enters the silicate flint glass of an equilateral prism as shown in Fig. 32–54. At what angle does each beam leave the prism (give angle with normal to the face)? See Fig. 32–28.

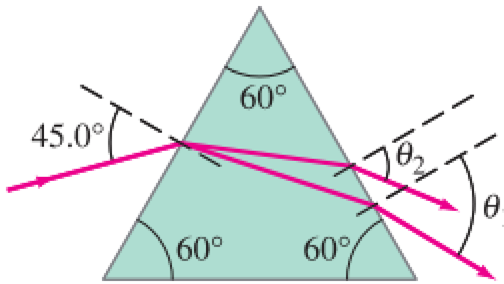


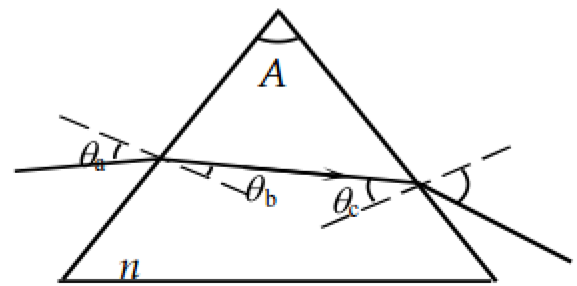
FIGURE 32–54 Problem 54.

54. The indices of refraction are estimated from Figure 32-28 as 1.642 for 465 nm and 1.619 for 652 nm. Consider the refraction at the first surface.

$$n_{air} \sin\theta_a = n \sin\theta_b \rightarrow$$

$$(1.00)\sin 45° = (1.642)\sin\theta_{b1} \rightarrow \theta_{b1} = 25.51°$$

$$(1.00)\sin 45° = (1.619)\sin\theta_{b2} \rightarrow \theta_{b2} = 25.90°$$



We find the angle of incidence at the second surface from the upper triangle.

$$(90° - \theta_b) + (90° - \theta_c) + A = 180° \rightarrow$$

$$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.51° = 34.49° \;;\; \theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°$$

Apply Snell's law at the second surface.

$$n \sin\theta_c = n_{air} \sin\theta_d$$

$$(1.642)\sin 34.49° = (1.00)\sin\theta_{d1} \rightarrow \boxed{\theta_{d1} = 68.4° \text{ from the normal}}$$

$$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2} \rightarrow \boxed{\theta_{d2} = 65.2° \text{ from the normal}}$$

61. (II) A beam of light is emitted 8.0 cm beneath the surface of a liquid and strikes the surface 7.6 cm from the point directly above the source. If total internal reflection occurs, what can you say about the index of refraction of the liquid?

61. We find the angle of incidence from the distances.

$$\tan\theta_1 = \frac{\ell}{h} = \frac{(7.6\,\text{cm})}{(8.0\,\text{cm})} = 0.95 \rightarrow \theta_1 = 43.53^\circ$$

The relationship for the maximum incident angle for refraction from liquid into air gives this.

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \rightarrow n_{\text{liquid}}\sin\theta_{1\,\text{max}} = (1.00)\sin 90^\circ \rightarrow \sin\theta_{1\,\text{max}} = \frac{1}{n_{\text{liquid}}}$$

Thus we have the following.

$$\sin\theta_1 \geq \sin\theta_{1\,\text{max}} = \frac{1}{n_{\text{liquid}}} \rightarrow \sin 43.53^\circ = 0.6887 \geq \frac{1}{n_{\text{liquid}}} \rightarrow \boxed{n_{\text{liquid}} \geq 1.5}$$

74. We wish to determine the depth of a swimming pool filled with water by measuring the width ($x = 5.50\,\text{m}$) and then noting that the bottom edge of the pool is just visible at an angle of 13.0° above the horizontal as shown in Fig. 32–62. Calculate the depth of the pool.

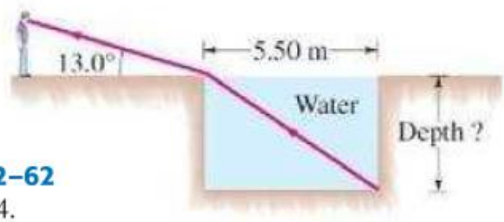


FIGURE 32–62 Problem 74.

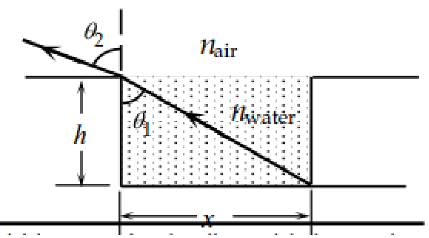
74. Find the angle of incidence for refraction from water into air.

$$n_{\text{water}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \rightarrow$$

$$(1.33)\sin\theta_1 = (1.00)\sin(90.0^\circ - 13.0^\circ) \rightarrow \theta_1 = 47.11^\circ$$

$$(1.33)\sin\theta_1 = (1.00)\sin(90.0^\circ - 13.0^\circ),$$

We find the depth of the pool from $\tan\theta_1 = x/h$.



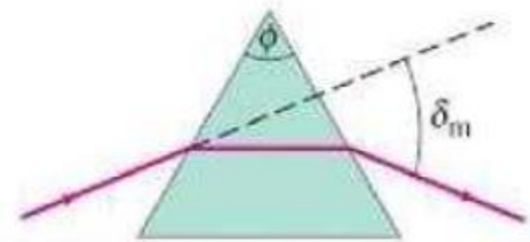
$$\tan 47.11^\circ = (5.50\,\text{m})/h \rightarrow h = \boxed{5.11\,\text{m}}$$

79. When light passes through a prism, the angle that the refracted ray makes relative to the incident ray is called the deviation angle δ, Fig. 32–64. Show that this angle is a minimum when the ray passes through the prism symmetrically, perpendicular to the bisector of the apex angle ϕ, and show that the minimum deviation angle, δ_m, is related to the prism's index of refraction n by

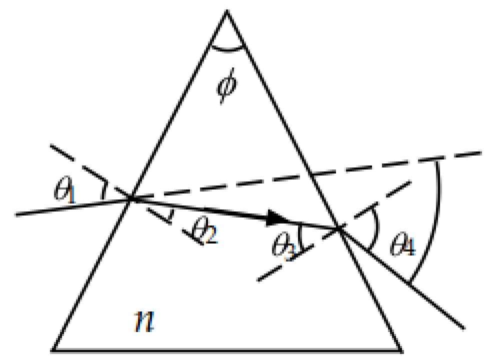
$$n = \frac{\sin\frac{1}{2}(\phi + \delta_m)}{\sin\phi/2}.$$

[*Hint*: For θ in radians, $(d/d\theta)(\sin^{-1}\theta) = 1/\sqrt{1-\theta^2}$.]

FIGURE 32–64 Problems 79 and 80.



79. The total deviation of the beam is the sum of the deviations at each surface. The deviation at the first surface is the refracted angle θ_2 subtracted from the incident angle θ_1. The deviation at the second surface is the incident angle θ_3 subtracted from the refracted angle θ_4. This gives the total deviation.



$$\delta = \delta_1 + \delta_2 = \theta_1 - \theta_2 + \theta_4 - \theta_3$$

We will express all of the angles in terms of θ_2. To minimize the deviation, we will take the derivative of the deviation with respect to θ_2, and then set that derivative equal to zero. Use Snell's law at the first surface to write the incident angle in terms of the refracted angle.

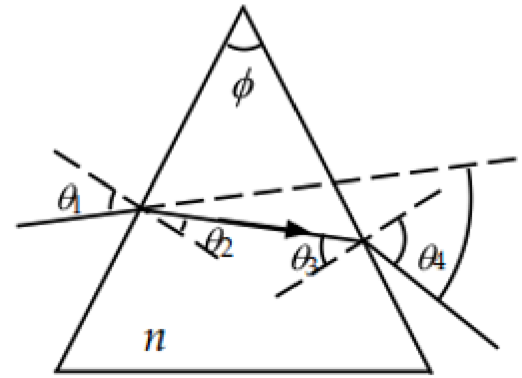
$$\sin\theta_1 = n\sin\theta_2 \quad \rightarrow \quad \theta_1 = \sin^{-1}(n\sin\theta_2)$$

The angle of incidence at the second surface is found using complementary angles, such that the sum of the refracted angle from the first surface and the incident angle at the second surface must equal the apex angle.

$$\phi = \theta_2 + \theta_3 \quad \rightarrow \quad \theta_3 = \phi - \theta_2$$

79. When light passes through a prism, the angle that the refracted ray makes relative to the incident ray is called the deviation angle δ, Fig. 32–64. Show that this angle is a minimum when the ray passes through the prism symmetrically, perpendicular to the bisector of the apex angle ϕ, and show that the minimum deviation angle, δ_m, is related to the prism's index of refraction n by

$$n = \frac{\sin\frac{1}{2}(\phi + \delta_m)}{\sin\phi/2}.$$



[*Hint*: For θ in radians, $(d/d\theta)(\sin^{-1}\theta) = 1/\sqrt{1-\theta^2}$.]

The refracted angle from the second surface is again found using Snell's law with the deviation in angle equal to the difference between the incident and refracted angles at the second surface.

$$n\sin\theta_3 = \sin\theta_4 \quad\rightarrow\quad \theta_4 = \sin^{-1}(n\sin\theta_3) = \sin^{-1}(n\sin(\phi-\theta_2))$$

Inserting each of the angles into the deviation and setting the derivative equal to zero allows us to solve for the angle at which the deviation is a minimum.

$$\delta = \sin^{-1}(n\sin\theta_2) - \theta_2 + \sin^{-1}(n\sin(\phi-\theta_2)) - (\phi-\theta_2)$$

$$= \sin^{-1}(n\sin\theta_2) + \sin^{-1}(n\sin(\phi-\theta_2)) - \phi$$

$$\frac{d\delta}{d\theta_2} = \frac{n\cos\theta_2}{\sqrt{1-n^2\sin^2\theta_2}} - \frac{n\cos(\phi-\theta_2)}{\sqrt{1-n^2\sin^2(\phi-\theta_2)}} = 0 \quad\rightarrow\quad \theta_2 = \phi-\theta_2 \quad\rightarrow\quad \theta_2 = \theta_3 = \tfrac{1}{2}\phi$$

In order for $\theta_2 = \theta_3$, the ray must pass through the prism horizontally, which is perpendicular to the bisector of the apex angle ϕ. Set $\theta_2 = \frac{1}{2}\phi$ in the deviation equation (for the minimum deviation, δ_m) and solve for the index of refraction.

$$\delta_m = \sin^{-1}(n\sin\theta_2) + \sin^{-1}(n\sin(\phi-\theta_2)) - \phi$$

$$= \sin^{-1}(n\sin\tfrac{1}{2}\phi) + \sin^{-1}(n\sin\tfrac{1}{2}\phi) - \phi = 2\sin^{-1}(n\sin\tfrac{1}{2}\phi) - \phi$$

$$\rightarrow \boxed{n = \frac{\sin\left(\frac{1}{2}(\delta_m+\phi)\right)}{\sin\frac{1}{2}\phi}}$$

86. The paint used on highway signs often contains small transparent spheres which provide nighttime illumination of the sign's lettering by retro-reflecting vehicle headlight beams. Consider a light ray from air incident on one such sphere of radius r and index of refraction n. Let θ be its incident angle, and let the ray follow the path shown in Fig. 32–66, so that the ray exits the sphere in the direction exactly antiparallel to its incoming direction. Considering only rays for which $\sin\theta$ can be approximated as θ, determine the required value for n.

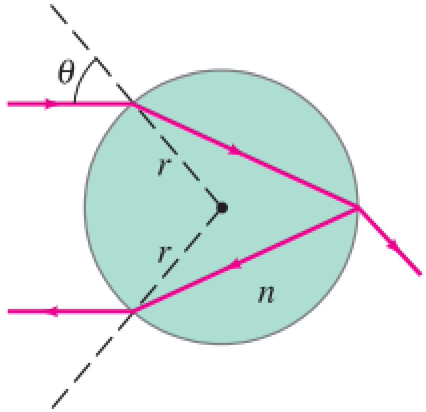


FIGURE 32–66 Problem 86.

86. The path of the ray in the sphere forms an isosceles triangle with two radii. The two identical angles of the triangle are equal to the refracted angle. Since the incoming ray is horizontal, the third angle is the supplementary angle of the incident angle. We set the sum of these angles equal to 180° and solve for the ratio of the incident and refracted angles. Finally we use Snell's law in the small angle approximation to calculate the index of refraction.

$$2\theta_r + (180° - \theta) = 180° \rightarrow \theta = 2\theta_r$$

$$n_1 \sin\theta = n_2 \sin\theta_r \rightarrow \theta = n\theta_r = 2\theta_r \rightarrow \boxed{n = 2}$$