1. Is the frequency of a simple periodic wave equal to the frequency of its source? Why or why not?

1. Yes. A simple periodic wave travels through a medium, which must be in contact with or connected to the source for the wave to be generated. If the medium changes, the wave speed and wavelength can change but the frequency remains constant.

4. What kind of waves do you think will travel down a horizontal metal rod if you strike its end (*a*) vertically from above and (*b*) horizontally parallel to its length?

4. (*a*) Striking the rod vertically from above will displace particles in a direction perpendicular to the rod and will set up primarily transverse waves.
   (*b*) Striking the rod horizontally parallel to its length will give the particles an initial displacement parallel to the rod and will set up primarily longitudinal waves.

7. The speed of sound in most solids is somewhat greater than in air, yet the density of solids is much greater (10^3 to 10^4 times). Explain.

7. The speed of sound is defined as $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of the material. The bulk modulus of most solids is at least 10^6 times as great as the bulk modulus of air. This difference overcomes the larger density of most solids, and accounts for the greater speed of sound in most solids than in air.

14. If a string is vibrating as a standing wave in three segments, are there any places you could touch it with a knife blade without disturbing the motion?

14. Yes. If you touch the string at any node you will not disturb the motion. There will be nodes at each end as well as at the points one-third and two-thirds of the distance along the length of the string.

1. (I) A fisherman notices that wave crests pass the bow of his anchored boat every 3.0 s. He measures the distance between two crests to be 8.0 m. How fast are the waves traveling?

1. The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 8.0 m.

$$v = \lambda f = \lambda / T = (8.0\,\text{m})/(3.0\,\text{s}) = \boxed{2.7\,\text{m/s}}$$

4. (I) AM radio signals have frequencies between 550 kHz and 1600 kHz (kilohertz) and travel with a speed of 3.0×10^8 m/s. What are the wavelengths of these signals? On FM the frequencies range from 88 MHz to 108 MHz (megahertz) and travel at the same speed. What are their wavelengths?

4. To find the wavelength, use $\lambda = v/f$.

AM: $$\lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\,\text{m/s}}{550\times10^3\,\text{Hz}} = 545\,\text{m} \qquad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\,\text{m/s}}{1600\times10^3\,\text{Hz}} = 188\,\text{m} \qquad \boxed{\text{AM: 190 m to 550 m}}$$

FM: $$\lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\,\text{m/s}}{88\times10^6\,\text{Hz}} = 3.41\,\text{m} \qquad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\,\text{m/s}}{108\times10^6\,\text{Hz}} = 2.78\,\text{m} \qquad \boxed{\text{FM: 2.8 m to 3.4 m}}$$

9. (II) A ski gondola is connected to the top of a hill by a steel cable of length 660 m and diameter 1.5 cm. As the gondola comes to the end of its run, it bumps into the terminal and sends a wave pulse along the cable. It is observed that it took 17 s for the pulse to return. (*a*) What is the speed of the pulse? (*b*) What is the tension in the cable?

9. (*a*) The speed of the pulse is given by

$$v = \frac{\Delta x}{\Delta t} = \frac{2(660\,\text{m})}{17\,\text{s}} = 77.65\,\text{m/s} \approx \boxed{78\,\text{m/s}}$$

(*b*) The tension is related to the speed of the pulse by $v = \sqrt{F_T/\mu}$. The mass per unit length of the cable can be found from its volume and density.

$$\rho = \frac{m}{V} = \frac{m}{\pi (d/2)^2 \ell} \rightarrow$$

$$\mu = \frac{m}{\ell} = \pi\rho\left(\frac{d}{2}\right)^2 = \pi\left(7.8\times10^3\,\text{kg/m}^3\right)\left(\frac{1.5\times10^{-2}\,\text{m}}{2}\right)^2 = 1.378\,\text{kg/m}$$

$$v = \sqrt{F_T/\mu} \rightarrow F_T = v^2\mu = (77.65\,\text{m/s})^2(1.378\,\text{kg/m}) = \boxed{8300\,\text{N}}$$

19. (II) A small steel wire of diameter 1.0 mm is connected to an oscillator and is under a tension of 7.5 N. The frequency of the oscillator is 60.0 Hz and it is observed that the amplitude of the wave on the steel wire is 0.50 cm. (*a*) What is the power output of the oscillator, assuming that the wave is not reflected back? (*b*) If the power output stays constant but the frequency is doubled, what is the amplitude of the wave?

19. (*a*) The power transmitted by the wave is assumed to be the same as the output of the oscillator. That power is given by Eq. 15-6. The wave speed is given by Eq. 15-2. Note that the mass per unit length can be expressed as the volume mass density times the cross sectional area.

$$\bar{P} = 2\pi^2 \rho S v f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_T}{\mu}} f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_T}{\rho S}} f^2 A^2 = 2\pi^2 f^2 A^2 \sqrt{S\rho F_T}$$

$$= 2\pi^2 (60.0\,\text{Hz})^2 (0.0050\,\text{m})^2 \sqrt{\pi (5.0\times10^{-3}\,\text{m})^2 (7800\,\text{kg/m}^3)(7.5\,\text{N})} = \boxed{0.38\,\text{W}}$$

(*b*) The frequency and amplitude are both squared in the equation. Thus is the power is constant, and the frequency doubles, the amplitude must be halved, and so be $\boxed{0.25\,\text{cm}}$.

22. (I) A transverse wave on a wire is given by $D(x,t) = 0.015\sin(25x - 1200t)$ where D and x are in meters and t is in seconds. (*a*) Write an expression for a wave with the same amplitude, wavelength, and frequency but traveling in the opposite direction. (*b*) What is the speed of either wave?

22. (a) The only difference is the direction of motion.

$$D(x,t) = 0.015\sin(25x + 1200t)$$

(*b*) The speed is found from the wave number and the angular frequency, Eq. 15-12.

$$v = \frac{\omega}{k} = \frac{1200\,\text{rad/s}}{25\,\text{rad/m}} = \boxed{48\,\text{m/s}}$$

23. (I) Suppose at $t = 0$, a wave shape is represented by $D = A\sin(2\pi x/\lambda + \phi)$; that is, it differs from Eq. 15–9 by a constant phase factor ϕ. What then will be the equation for a wave traveling to the left along the x axis as a function of x and t?

23. To represent a wave traveling to the left, we replace x by $x + vt$. The resulting expression can be given in various forms.

$$\boxed{D = A\sin\left[2\pi(x+vt)/\lambda+\phi\right] = A\sin\left[2\pi\left(\frac{x}{\lambda}+\frac{vt}{\lambda}\right)+\phi\right] = A\sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)+\phi\right] = A\sin(kx+\omega t+\phi)}$$

26. (II) A transverse wave on a cord is given by $D(x, t) = 0.12\sin(3.0x - 15.0t)$, where D and x are in m and t is in s. At $t = 0.20\text{ s}$, what are the displacement and velocity of the point on the cord where $x = 0.60\text{ m}$?

26. The displacement of a point on the cord is given by the wave, $D(x,t) = 0.12\sin(3.0x - 15.0t)$. The velocity of a point on the cord is given by $\frac{\partial D}{\partial t}$.

$$D(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})\sin\left[(3.0\,\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.11\,\text{m}}$$

$$\frac{\partial D}{\partial t} = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos(3.0x - 15.0t)$$

$$\frac{\partial D}{\partial t}(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos\left[(3.0\,\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.65\,\text{m/s}}$$

*31. (II) Determine if the function $D = A \sin kx \cos \omega t$ is a solution of the wave equation.

31. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D = A \sin kx \cos \omega t$$

$$\frac{\partial D}{\partial x} = kA \cos kx \cos \omega t \ ; \ \frac{\partial^2 D}{\partial x^2} = -k^2 A \sin kx \cos \omega t$$

$$\frac{\partial D}{\partial t} = -\omega A \sin kx \sin \omega t \ ; \ \frac{\partial^2 D}{\partial t^2} = -\omega^2 A \sin kx \cos \omega t$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 D}{\partial t^2}$, and since $v = \frac{\omega}{k}$ from Eq. 15-12, we have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

Yes, the function is a solution.

*35. (II) Does the function $D(x,t) = e^{-(kx-\omega t)^2}$ satisfy the wave equation? Why or why not?

35. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D = e^{-(kx-\omega t)^2} \ ; \ \frac{\partial D}{\partial x} = -2k(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2} = -2k(kx-\omega t)\left[-2k(kx-\omega t)e^{-(kx-\omega t)^2}\right] + (-2k^2)e^{-(kx-\omega t)^2} = 2k^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial D}{\partial t} = 2\omega(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial t^2} = 2\omega(kx-\omega t)\left[2\omega(kx-\omega t)e^{-(kx-\omega t)^2}\right] + (-2\omega^2)e^{-(kx-\omega t)^2} = 2\omega^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} \rightarrow 2k^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2} = \frac{1}{v^2}2\omega^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2} \rightarrow$$

$$k^2 = \frac{\omega^2}{v^2}$$

Since $v = \frac{\omega}{k}$, we have an identity. Yes, the function is a solution.

38. (II) Consider a sine wave traveling down the stretched two-part cord of Fig. 15–19. Determine a formula (*a*) for the ratio of the speeds of the wave in the two sections, v_H/v_L, and (*b*) for the ratio of the wavelengths in the two sections. (The frequency is the same in both sections. Why?) (*c*) Is the wavelength larger in the heavier cord or the lighter?

38. (*a*) The speed of the wave in a stretched cord is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The tensions must be the same in both parts of the cord. If they were not the same, then the net longitudinal force on the joint between the two parts would not be zero, and the joint would have to accelerate along the length of the cord.

$$v = \sqrt{F_T/\mu} \rightarrow \frac{v_H}{v_L} = \frac{\sqrt{F_T/\mu_H}}{\sqrt{F_T/\mu_L}} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*b*) The frequency must be the same in both sections. If it were not, then the joint between the two sections would not be able to keep the two sections together. The ends could not stay in phase with each other if the frequencies were different.

$$f = \frac{v}{\lambda} \rightarrow \frac{v_H}{\lambda_H} = \frac{v_L}{\lambda_L} \rightarrow \frac{\lambda_H}{\lambda_L} = \frac{v_H}{v_L} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*c*) The ratio under the square root sign is less than 1, and so the lighter cord has the greater wavelength.

48. (II) The velocity of waves on a string is 96 m/s. If the frequency of standing waves is 445 Hz, how far apart are the two adjacent nodes?

48. Adjacent nodes are separated by a half-wavelength, as examination of Figure 15-26 will show.

$$\lambda = \frac{v}{f} \quad \rightarrow \quad \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{96\,\text{m/s}}{2(445\,\text{Hz})} = \boxed{0.11\,\text{m}}$$

51. (II) Show that the frequency of standing waves on a cord of length ℓ and linear density μ, which is stretched to a tension F_T, is given by

$$f = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}$$

where n is an integer.

51. The speed of the wave is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The wavelength of the fundamental is $\lambda_1 = 2\ell$. Thus the frequency of the fundamental is $f_1 = \frac{v}{\lambda_1} = \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu}}$. Each harmonic is present in a vibrating string, and so $f_n = nf_1 = \boxed{\frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}}$, $n = 1, 2, 3, \ldots$.

63. (II) Two oppositely directed traveling waves given by $D_1 = (5.0\,\text{mm})\cos[(2.0\,\text{m}^{-1})x - (3.0\,\text{rad/s})t]$ and $D_2 = (5.0\,\text{mm})\cos[(2.0\,\text{m}^{-1})x + (3.0\,\text{rad/s})t]$ form a standing wave. Determine the position of nodes along the x axis.

63. The standing wave is the sum of the two individual standing waves. We use the trigonometric identities for the cosine of a difference and a sum.

$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \;;\; \cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$

$$D = D_1 + D_2 = A\cos(kx - \omega t) + A\cos(kx + \omega t) = A[\cos(kx - \omega t) + \cos(kx + \omega t)]$$

$$= A[\cos kx \cos\omega t + \sin kx \sin\omega t + \cos kx \cos\omega t - \sin kx \sin\omega t]$$

$$= 2A\cos kx \cos\omega t$$

Thus the standing wave is $D = 2A\cos kx \cos\omega t$. The nodes occur where the position term forces $D = 2A\cos kx \cos\omega t = 0$ for all time. Thus $\cos kx = 0 \;\rightarrow\; kx = \pm(2n+1)\frac{\pi}{2},\ n = 0, 1, 2, \cdots$. Thus, since $k = 2.0\,\text{m}^{-1}$, we have $\boxed{x = \pm\left(n + \tfrac{1}{2}\right)\frac{\pi}{2}\,\text{m},\ n = 0, 1, 2, \cdots}$.

78. A uniform cord of length ℓ and mass m is hung vertically from a support. (*a*) Show that the speed of transverse waves in this cord is $\sqrt{gh}$, where h is the height above the lower end. (*b*) How long does it take for a pulse to travel upward from one end to the other?

78. (*a*) The speed of the wave at a point h above the lower end depends on the tension at that point and the linear mass density of the cord. The tension must equal the mass of the lower segment if the lower segment is in equilibrium. Use Eq. 15-2 for the wave speed.

$$F_{\text{T}} = m_{\text{segment}}g = \frac{h}{\ell}mg \ ; \ v = \sqrt{\frac{F_{\text{T}}}{\mu}} = \sqrt{\frac{\frac{h}{\ell}mg}{\frac{m}{\ell}}} = \boxed{\sqrt{hg}}$$

(*b*) We treat h as a variable, measured from the bottom of the cord. The wave speed at that point is given above as $v = \sqrt{hg}$. The distance a wave would travel up the cord during a time dt is then $dh = vdt = \sqrt{hg}\,dt$. To find the total time for a wave to travel up the cord, integrate over the length of the cord.

$$dh = vdt = \sqrt{hg}dt \ \rightarrow \ dt = \frac{dh}{\sqrt{hg}} \ \rightarrow \ \int_0^{t_{\text{total}}} dt = \int_0^L \frac{dh}{\sqrt{hg}} \ \rightarrow$$

$$t_{\text{total}} = \int_0^L \frac{dh}{\sqrt{hg}} = 2\sqrt{\frac{h}{g}}\Bigg|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

84. A wave with a frequency of 220 Hz and a wavelength of 10.0 cm is traveling along a cord. The maximum speed of particles on the cord is the same as the wave speed. What is the amplitude of the wave?

84. We take the wave function to be $D(x,t) = A\sin(kx - \omega t)$. The wave speed is given by $v = \frac{\omega}{k} = \frac{\lambda}{f}$, while the speed of particles on the cord is given by $\frac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t} = -\omega A\cos(kx - \omega t) \quad \rightarrow \quad \left(\frac{\partial D}{\partial t}\right)_{\max} = \omega A$$

$$\omega A = v = \frac{\omega}{k} \quad \rightarrow \quad A = \frac{1}{k} = \frac{\lambda}{2\pi} = \frac{10.0\,\text{cm}}{2\pi} = \boxed{1.59\,\text{cm}}$$