9. Two linear waves have the same amplitude and speed, and otherwise are identical, except one has half the wavelength of the other. Which transmits more energy? By what factor?

9. If two waves have the same speed but one has half the wavelength of the other, the wave with the shorter wavelength must have twice the frequency of the other. The energy transmitted by a wave depends on the wave speed and the square of the frequency. The wave with the shorter wavelength will transmit four times the energy transmitted by the other wave.

16. Can the amplitude of the standing waves in Fig. 15–25 be greater than the amplitude of the vibrations that cause them (up and down motion of the hand)?

16. Yes. A standing wave is an example of a resonance phenomenon, caused by constructive interference between a traveling wave and its reflection. The wave energy is distributed around the antinodes, which exhibit large amplitude oscillations, even when the generating oscillations from the hand are small.

* **19.** If we knew that energy was being transmitted from one place to another, how might we determine whether the energy was being carried by particles (material objects) or by waves?

19. Waves exhibit diffraction. If a barrier is placed between the energy source and the energy receiver, and energy is still received, it is a good indication that the energy is being carried by waves. If placement of the barrier stops the energy transfer, it may be because the energy is being transferred by particles or that the energy is being transferred by waves with wavelengths smaller than the barrier.

8. (II) A sailor strikes the side of his ship just below the surface of the sea. He hears the echo of the wave reflected from the ocean floor directly below 2.8 s later. How deep is the ocean at this point?

8. The speed of the water wave is given by $v=\sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 12-1, and ρ is the density of sea water, from Table 13-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v=\frac{2\ell}{t} \rightarrow \ell=\frac{vt}{2}=\frac{t}{2}\sqrt{\frac{B}{\rho}}=\frac{2.8\mathrm{s}}{2}\sqrt{\frac{2.0\times10^{9}\,\mathrm{N/m^2}}{1.025\times10^{3}\,\mathrm{kg/m^3}}}=\boxed{2.0\times10^{3}\,\mathrm{m}}$$

10. (II) P and S waves from an earthquake travel at different speeds, and this difference helps locate the earthquake "epicenter" (where the disturbance took place). (*a*) Assuming typical speeds of 8.5 km/s and 5.5 km/s for P and S waves, respectively, how far away did the earthquake occur if a particular seismic station detects the arrival of these two types of waves 1.7 min apart? (*b*) Is one seismic station sufficient to determine the position of the epicenter? Explain.

10. (*a*) Both waves travel the same distance, so $\Delta x = v_1 t_1 = v_2 t_2$. We let the smaller speed be v_1, and the larger speed be v_2. The slower wave will take longer to arrive, and so t_1 is more than t_2.

$$t_1 = t_2 + 1.7\text{ min} = t_2 + 102\text{ s} \rightarrow v_1(t_2 + 102\text{ s}) = v_2 t_2 \rightarrow$$

$$t_2 = \frac{v_1}{v_2 - v_1}(102\text{ s}) = \frac{5.5\text{ km/s}}{8.5\text{ km/s} - 5.5\text{km/s}}(102\text{ s}) = 187\text{ s}$$

$$\Delta x = v_2 t_2 = (8.5\text{ km/s})(187\text{ s}) = \boxed{1600\text{ km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius 1.9×10^3 km from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

19. (II) A small steel wire of diameter 1.0 mm is connected to an oscillator and is under a tension of 7.5 N. The frequency of the oscillator is 60.0 Hz and it is observed that the amplitude of the wave on the steel wire is 0.50 cm. (*a*) What is the power output of the oscillator, assuming that the wave is not reflected back? (*b*) If the power output stays constant but the frequency is doubled, what is the amplitude of the wave?

19. (*a*) The power transmitted by the wave is assumed to be the same as the output of the oscillator. That power is given by Eq. 15-6. The wave speed is given by Eq. 15-2. Note that the mass per unit length can be expressed as the volume mass density times the cross sectional area.

$$\bar{P} = 2\pi^2 \rho S v f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_T}{\mu}} f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_T}{\rho S}} f^2 A^2 = 2\pi^2 f^2 A^2 \sqrt{S \rho F_T}$$

$$= 2\pi^2 (60.0\,\text{Hz})^2 (0.0050\,\text{m})^2 \sqrt{\pi (5.0\times10^{-3}\,\text{m})^2 (7800\,\text{kg/m}^3)(7.5\,\text{N})} = \boxed{0.38\,\text{W}}$$

(*b*) The frequency and amplitude are both squared in the equation. Thus is the power is constant, and the frequency doubles, the amplitude must be halved, and so be $\boxed{0.25\,\text{cm}}$.

*35. (II) Does the function $D(x,t) = e^{-(kx-\omega t)^2}$ satisfy the wave equation? Why or why not?

35. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D = e^{-(kx-\omega t)^2} \ ; \ \frac{\partial D}{\partial x} = -2k(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2} = -2k(kx-\omega t)\left[-2k(kx-\omega t)e^{-(kx-\omega t)^2}\right] + \left(-2k^2\right)e^{-(kx-\omega t)^2} = 2k^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial D}{\partial t} = 2\omega(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial t^2} = 2\omega(kx-\omega t)\left[2\omega(kx-\omega t)e^{-(kx-\omega t)^2}\right] + \left(-2\omega^2\right)e^{-(kx-\omega t)^2} = 2\omega^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} \rightarrow 2k^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2} = \frac{1}{v^2}2\omega^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2} \rightarrow$$

$$k^2 = \frac{\omega^2}{v^2}$$

Since $v = \frac{\omega}{k}$, we have an identity. Yes, the function is a solution.

51. (II) Show that the frequency of standing waves on a cord of length ℓ and linear density μ, which is stretched to a tension F_T, is given by

$$f = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}$$

where n is an integer.

51. The speed of the wave is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The wavelength of the fundamental is $\lambda_1 = 2\ell$. Thus the frequency of the fundamental is $f_1 = \frac{v}{\lambda_1} = \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu}}$. Each harmonic is present in a vibrating string, and so $f_n = nf_1 = \boxed{\frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}}$, $n = 1, 2, 3, \ldots$.

71. A sinusoidal traveling wave has frequency 880 Hz and phase velocity 440 m/s. (*a*) At a given time, find the distance between any two locations that correspond to a difference in phase of $\pi/6$ rad. (*b*) At a fixed location, by how much does the phase change during a time interval of 1.0×10^{-4} s?

71. The frequency is 880 Hz and the phase velocity is 440 m/s, so the wavelength is

$$\lambda = \frac{v}{f} = \frac{440\,\text{m/s}}{880\ \text{Hz}} = 0.50\,\text{m}.$$

(*a*) Use the ratio of distance to wavelength to define the phase difference.

$$\frac{x}{\lambda} = \frac{\pi/6}{2\pi} \quad \rightarrow \quad x = \frac{\lambda}{12} = \frac{0.50\,\text{m}}{12} = \boxed{0.042\,\text{m}}$$

(b) Use the ratio of time to period to define the phase difference.

$$\frac{t}{T} = \frac{\phi}{2\pi} \quad \rightarrow \quad \phi = \frac{2\pi t}{T} = 2\pi t f = 2\pi\left(1.0\times10^{-4}\,\text{s}\right)\left(880\,\text{Hz}\right) = \boxed{0.55\,\text{rad}}$$

78. A uniform cord of length ℓ and mass m is hung vertically from a support. (*a*) Show that the speed of transverse waves in this cord is $\sqrt{gh}$, where h is the height above the lower end. (*b*) How long does it take for a pulse to travel upward from one end to the other?

78. (*a*) The speed of the wave at a point h above the lower end depends on the tension at that point and the linear mass density of the cord. The tension must equal the mass of the lower segment if the lower segment is in equilibrium. Use Eq. 15-2 for the wave speed.

$$F_T = m_{\text{segment}}g = \frac{h}{\ell}mg \;;\; v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\frac{h}{\ell}mg}{\frac{m}{\ell}}} = \boxed{\sqrt{hg}}$$

(*b*) We treat h as a variable, measured from the bottom of the cord. The wave speed at that point is given above as $v = \sqrt{hg}$. The distance a wave would travel up the cord during a time dt is then $dh = vdt = \sqrt{hg}\,dt$. To find the total time for a wave to travel up the cord, integrate over the length of the cord.

$$dh = vdt = \sqrt{hg}dt \;\rightarrow\; dt = \frac{dh}{\sqrt{hg}} \;\rightarrow\; \int_0^{t_{\text{total}}} dt = \int_0^L \frac{dh}{\sqrt{hg}} \;\rightarrow$$

$$t_{\text{total}} = \int_0^L \frac{dh}{\sqrt{hg}} = 2\sqrt{\frac{h}{g}}\Bigg|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

91. For a spherical wave traveling uniformly away from a point source, show that the displacement can be represented by

$$D = \left(\frac{A}{r}\right)\sin(kr - \omega t),$$

where r is the radial distance from the source and A is a constant.

91. Because the radiation is uniform, the same energy must pass through every spherical surface, which has the surface area $4\pi r^2$. Thus the intensity must decrease as $1/r^2$. Since the intensity is proportional to the square of the amplitude, the amplitude will decrease as $1/r$. The radial motion will be sinusoidal, and so we have $D = \left(\frac{A}{r}\right)\sin(kr - \omega t)$.