2. Is the acceleration of a simple harmonic oscillator ever zero? If so, where?

2. The acceleration of a simple harmonic oscillator is momentarily zero as the mass passes through the equilibrium point. At this point, there is no force on the mass and therefore no acceleration.

7. If a pendulum clock is accurate at sea level, will it gain or lose time when taken to high altitude? Why?

7. At high altitude, g is slightly smaller than it is at sea level. If g is smaller, then the period T of the pendulum clock will be longer, and the clock will run slow (or lose time).

12. Does a car bounce on its springs faster when it is empty or when it is fully loaded?

12. Empty. The period of the oscillation of a spring increases with increasing mass, so when the car is empty the period of the harmonic motion of the springs will be shorter, and the car will bounce faster.

17. Why can you make water slosh back and forth in a pan only if you shake the pan at a certain frequency?

17. If you shake the pan at a resonant frequency, standing waves will be set up in the water and it will slosh back and forth. Shaking the pan at other frequencies will not create large waves. The individual water molecules will move but not in a coherent way.

10. (II) A mass m at the end of a spring oscillates with a frequency of 0.83 Hz. When an additional 680-g mass is added to m, the frequency is 0.60 Hz. What is the value of m?

10. The spring constant is the same regardless of what mass is attached to the spring.

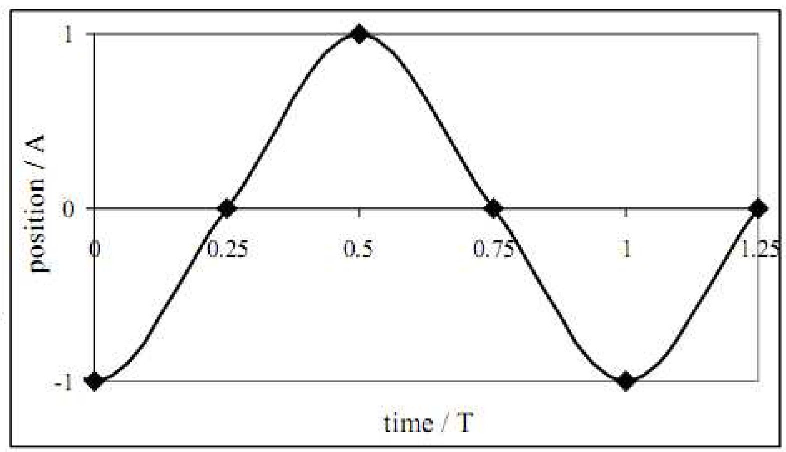
$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{k}{4\pi^2}=mf^2=\text{constant} \rightarrow m_1f_1^2=m_2f_1^2 \rightarrow$$

$$(m\ \text{kg})(0.83\,\text{Hz})^2=(m\ \text{kg}+0.68\ \text{kg})(0.60\ \text{Hz})^2 \rightarrow m=\frac{(0.68\,\text{kg})(0.60\,\text{Hz})^2}{(0.83\,\text{Hz})^2-(0.60\,\text{Hz})^2}=\boxed{0.74\,\text{kg}}$$

8. (II) Construct a Table indicating the position x of the mass in Fig. 14–2 at times $t=0,\frac{1}{4}T,\frac{1}{2}T,\frac{3}{4}T,T,$ and $\frac{5}{4}T$, where T is the period of oscillation. On a graph of x vs. t, plot these six points. Now connect these points with a smooth curve. Based on these simple considerations, does your curve resemble that of a cosine or sine wave?

8. The table of data is shown, along with the smoothed graph. Every quarter of a period, the mass moves from an extreme point to the equilibrium. The graph resembles a cosine wave (actually, the opposite of a cosine wave).

time	position
0	- A
T/4	0
T/2	A
3T/4	0
T	- A
5T/4	0



16. (II) The graph of displacement vs. time for a small mass m at the end of a spring is shown in Fig. 14–30. At $t = 0$, $x = 0.43$ cm. (*a*) If $m = 9.5$ g, find the spring constant, k. (*b*) Write the equation for displacement x as a function of time.

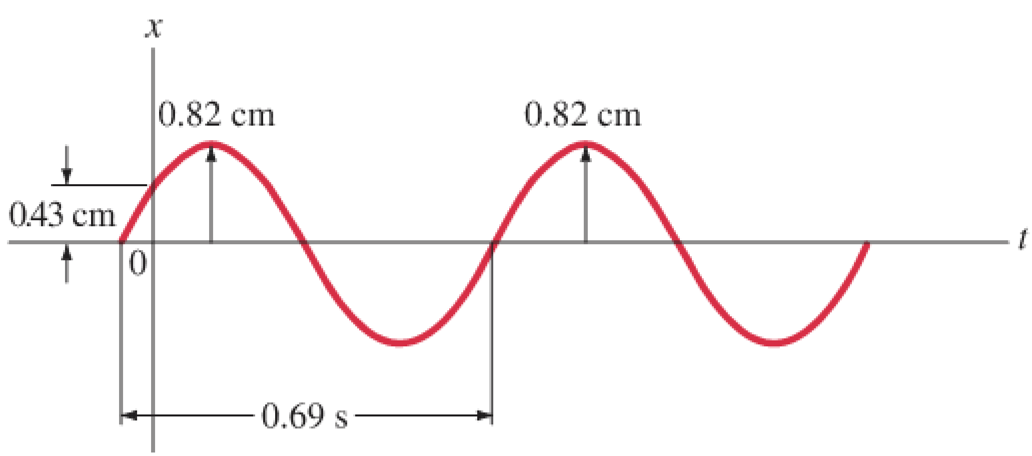


FIGURE 14–30 Problem 16.

16. (*a*) From the graph, the period is 0.69 s. The period and the mass can be used to find the spring constant.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = 4\pi^2 \frac{m}{T^2} = 4\pi^2 \frac{0.0095\,\text{kg}}{(0.69\,\text{s})^2} = 0.7877\,\text{N/m} \approx \boxed{0.79\,\text{N/m}}$$

(*b*) From the graph, the amplitude is 0.82 cm. The phase constant can be found from the initial conditions.

$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right) = (0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t + \phi\right)$$

$$x(0) = (0.82\,\text{cm})\cos\phi = 0.43\,\text{cm} \quad \rightarrow \quad \phi = \cos^{-1}\frac{0.43}{0.82} = \pm 1.02\,\text{rad}$$

Because the graph is shifted to the RIGHT from the 0-phase cosine, the phase constant must be subtracted.

$$x = \boxed{(0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t - 1.0\right)} \text{ or } (0.82\,\text{cm})\cos(9.1t - 1.0)$$

length of the bungee cord assuming SHM.

24. (II) A block of mass m is supported by two identical parallel vertical springs, each with spring stiffness constant k (Fig. 14–31). What will be the frequency of vertical oscillation?

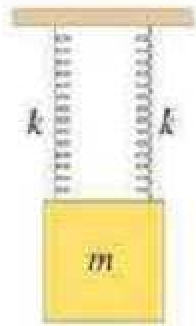
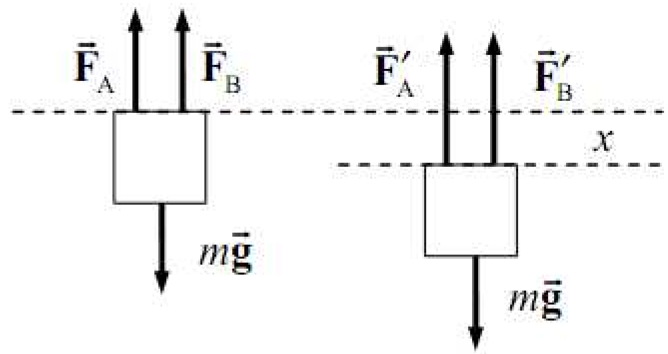


FIGURE 14–31
Problem 24.

ΜΟΡΙΟ ΜΕ ΔΙΠΛΟ ΔΕΣΜΟ!

24. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's second law for vertical forces, with up as positive, gives this.



$$\sum F_y = F_A + F_B - mg = 0 \quad \rightarrow \quad F_A + F_B = mg$$

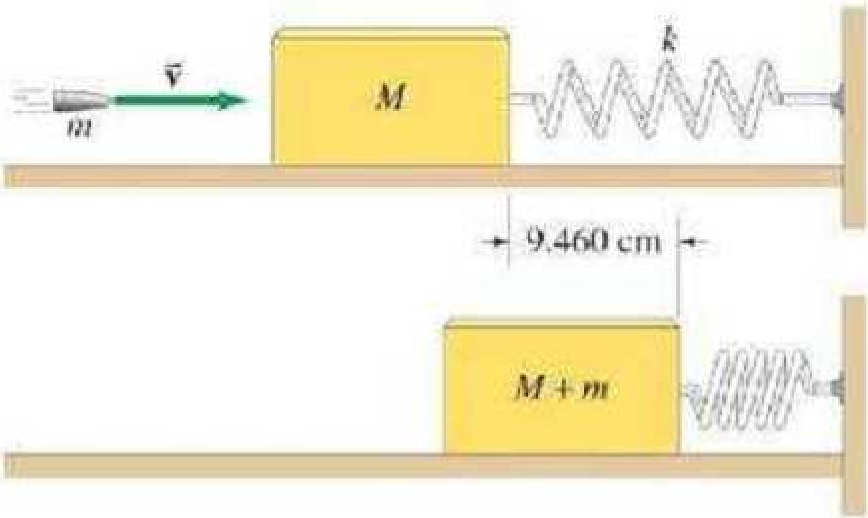
Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x < 0$. Again write Newton's second law for vertical forces.

$$\sum F_y = F_{net} = F'_A + F'_B - mg = F_A - kx + F_B - kx - mg = -2kx + (F_A + F_B - mg) = -2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows.

$$f = \frac{1}{2\pi}\sqrt{k_{\text{effective}}/m} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

37. (II) Agent Arlene devised the following method of measuring the muzzle velocity of a rifle (Fig. 14–33). She fires a bullet into a 4.648-kg wooden block resting on a smooth surface, and attached to a spring of spring constant $k = 142.7\,\text{N/m}$. The bullet, whose mass is 7.870 g, remains embedded in the wooden block. She measures the maximum distance that the block compresses the spring to be 9.460 cm. What is the speed v of the bullet?



37. We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then, the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$p_{\text{before}} = p_{\text{after}} \rightarrow mv_0 = (m+M)v_{\max} \rightarrow v_{\max} = \frac{m}{m+M}v_0$$

$$\tfrac{1}{2}(m+M)v_{\max}^2 = \tfrac{1}{2}kA^2 \rightarrow \tfrac{1}{2}(m+M)\left(\frac{m}{m+M}v_0\right)^2 = \tfrac{1}{2}kA^2 \rightarrow$$

$$v_0 = \frac{A}{m}\sqrt{k(m+M)} = \frac{(9.460\times10^{-2}\,\text{m})}{(7.870\times10^{-3}\,\text{kg})}\sqrt{(142.7\,\text{N/m})(7.870\times10^{-3}\,\text{kg}+4.648\,\text{kg})}$$

$$= \boxed{309.8\,\text{m/s}}$$

*52. (II) A student wants to use a meter stick as a pendulum. She plans to drill a small hole through the meter stick and suspend it from a smooth pin attached to the wall (Fig. 14–34). Where in the meter stick should she drill the hole to obtain the shortest possible period? How short an oscillation period can she obtain with a meter stick in this way?

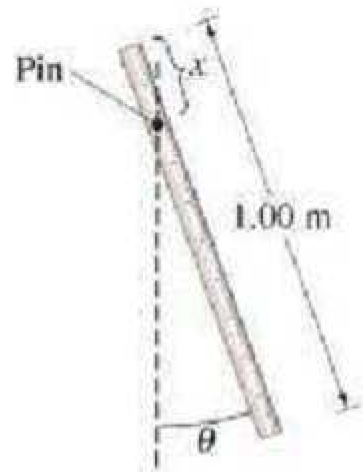


FIGURE 14–34
Problem 52.

52. The meter stick used as a pendulum is a physical pendulum. The period is given by Eq. 14-14, $T = 2\pi\sqrt{\frac{I}{mgh}}$. Use the parallel axis theorem to find the moment of inertia about the pin. Express the distances from the center of mass.

$$I = I_{CM} + mh^2 = \tfrac{1}{12}m\ell^2 + mh^2 \;\rightarrow\; T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{12}m\ell^2 + mh^2}{mgh}} = \frac{2\pi}{\sqrt{g}}\left(\tfrac{1}{12}\frac{\ell^2}{h} + h\right)^{1/2}$$

$$\frac{dT}{dh} = 2\pi\left(\tfrac{1}{2}\right)\left(\tfrac{1}{12}\frac{\ell^2}{h} + h\right)^{-1/2}\left(-\tfrac{1}{12}\frac{\ell^2}{h^2} + 1\right) = 0 \;\rightarrow\; h = \sqrt{\tfrac{1}{12}}\,\ell = 0.2887\,\text{m}$$

$$x = \tfrac{1}{2}\ell - h = 0.500 - 0.2887 \approx \boxed{0.211\,\text{m}} \text{ from the end}$$

Use the distance for h to calculate the period.

$$T = \frac{2\pi}{\sqrt{g}}\left(\tfrac{1}{12}\frac{\ell^2}{h} + h\right)^{1/2} = \frac{2\pi}{\sqrt{9.80\,\text{m/s}^2}}\left(\tfrac{1}{12}\frac{(1.00\,\text{m})^2}{0.2887\,\text{m}} + 0.2887\,\text{m}\right)^{1/2} = \boxed{1.53\,\text{s}}$$

56. (II) A 0.835-kg block oscillates on the end of a spring whose spring constant is $k = 41.0\,\mathrm{N/m}$. The mass moves in a fluid which offers a resistive force $F = -bv$, where $b = 0.662\,\mathrm{N\cdot s/m}$. (*a*) What is the period of the motion? (*b*) What is the fractional decrease in amplitude per cycle? (*c*) Write the displacement as a function of time if at $t = 0$, $x = 0$, and at $t = 1.00\,\mathrm{s}$, $x = 0.120\,\mathrm{m}$.

56. (*a*) The period of the motion can be found from Eq. 14-18, giving the angular frequency for the damped motion.

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{(41.0\,\mathrm{N/m})}{(0.835\,\mathrm{kg})} - \frac{(0.662\,\mathrm{N\cdot s/m})^2}{4(0.835\,\mathrm{kg})^2}} = 6.996\,\mathrm{rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6.996\,\mathrm{rad/s}} = \boxed{0.898\,\mathrm{s}}$$

(*b*) If the amplitude at some time is A, then one cycle later, the amplitude will be $Ae^{-\gamma T}$. Use this to find the fractional change.

$$\text{fractional change} = \frac{Ae^{-\gamma T} - A}{A} = e^{-\gamma T} - 1 = e^{-\frac{b}{2m}T} - 1 = e^{-\frac{(0.662\,\mathrm{N\cdot s/m})}{2(0.835\,\mathrm{kg})}(0.898\,\mathrm{s})} - 1 = \boxed{-0.300}$$

And so the amplitude decreases by 30% from the previous amplitude, every cycle.

And so the amplitude decreases by 30% from the previous amplitude, every cycle.

(c) Since the object is at the origin at $t = 0$, we will use a sine function to express the equation of motion.

$$x = Ae^{-\gamma t}\sin(\omega' t) \rightarrow 0.120\,\text{m} = Ae^{-\frac{(0.662\,\text{N}\cdot\text{s/m})}{2(0.835\,\text{kg})}(1.00\,\text{s})}\sin(6.996\,\text{rad}) \rightarrow$$

$$A = \frac{0.120\,\text{m}}{e^{-\frac{(0.662\,\text{N}\cdot\text{s/m})}{2(0.835\,\text{kg})}(1.00\,\text{s})}\sin(6.996\,\text{rad})} = 0.273\,\text{m} \;;\; \gamma = \frac{b}{2m} = \frac{(0.662\,\text{N}\cdot\text{s/m})}{2(0.835\,\text{kg})} = 0.396\,\text{s}^{-1}$$

$$\boxed{x = (0.273\,\text{m})e^{-(0.396\,\text{s}^{-1})t}\sin[(7.00\,\text{rad/s})t]}$$

mate the damping constant b. Neglect buoyant forces.

61. (III) (*a*) Show that the total mechanical energy, $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, as a function of time for a lightly damped harmonic oscillator is

$$E = \tfrac{1}{2}kA^2e^{-(b/m)t} = E_0e^{-(b/m)t},$$

where E_0 is the total mechanical energy at $t = 0$. (Assume $\omega' \gg b/2m$.) (*b*) Show that the fractional energy lost per period is

$$\frac{\Delta E}{E} = \frac{2\pi b}{m\omega_0} = \frac{2\pi}{Q},$$

where $\omega_0 = \sqrt{k/m}$ and $Q = m\omega_0/b$ is called the **quality factor** or **Q value** of the system. A larger Q value means the system can undergo oscillations for a longer time.

61. (*a*) For the "lightly damped" harmonic oscillator, we have $b^2 \ll 4mk \rightarrow \frac{b^2}{4m^2} \ll \frac{k}{m} \rightarrow \omega' \approx \omega_0$.

We also assume that the object starts to move from maximum displacement, and so

$$x = A_0 e^{-\frac{bt}{2m}} \cos\omega' t \text{ and } v = \frac{dx}{dt} = -\frac{b}{2m} A_0 e^{-\frac{bt}{2m}} \cos\omega' t - \omega' A_0 e^{-\frac{bt}{2m}} \sin\omega' t \approx -\omega_0 A_0 e^{-\frac{bt}{2m}} \sin\omega' t.$$

$$E = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 = \tfrac{1}{2}kA_0^2 e^{-\frac{bt}{m}} \cos^2\omega' t + \tfrac{1}{2}m\omega_0^2 A_0^2 e^{-\frac{bt}{m}} \sin^2\omega' t$$

$$= \tfrac{1}{2}kA_0^2 e^{-\frac{bt}{m}} \cos^2\omega' t + \tfrac{1}{2}kA_0^2 e^{-\frac{bt}{m}} \sin^2\omega' t = \tfrac{1}{2}kA_0^2 e^{-\frac{bt}{m}} = \boxed{E_0 e^{-\frac{bt}{m}}}$$

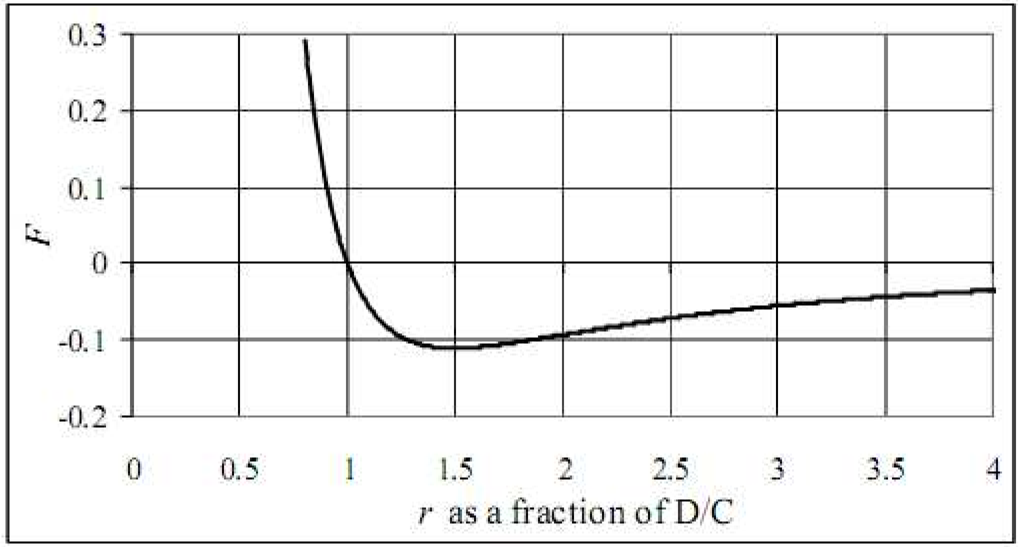
(*b*) The fractional loss of energy during one period is as follows. Note that we use the approximation that $\frac{b}{2m} \ll \omega_0 = \frac{2\pi}{T} \rightarrow \frac{bT}{m} \ll 4\pi \rightarrow \frac{bT}{m} \ll 1$.

$$\Delta E = E(t) - E(t+T) = E_0 e^{-\frac{bt}{m}} - E_0 e^{-\frac{b(t+T)}{m}} = E_0 e^{-\frac{bt}{m}} \left(1 - e^{-\frac{bT}{m}}\right) \rightarrow$$

$$\frac{\Delta E}{E} = \frac{E_0 e^{-\frac{bt}{m}} \left(1 - e^{-\frac{bT}{m}}\right)}{E_0 e^{-\frac{bt}{m}}} = 1 - e^{-\frac{bT}{m}} \approx 1 - \left(1 - \frac{bT}{m}\right) = \frac{bT}{m} = \frac{b2\pi}{m\omega_0} = \boxed{\frac{2\pi}{Q}}$$

84. In some diatomic molecules, the force each atom exerts on the other can be approximated by $F = -C/r^2 + D/r^3$, where r is the atomic separation and C and D are positive constants. (*a*) Graph F vs. r from $r = 0.8D/C$ to $r = 4D/C$. (*b*) Show that equilibrium occurs at $r = r_0 = D/C$. (*c*) Let $\Delta r = r - r_0$ be a small displacement from equilibrium, where $\Delta r \ll r_0$. Show that for such small displacements, the motion is approximately simple harmonic, and (*d*) determine the force constant. (*e*) What is the period of such motion? [*Hint*: Assume one atom is kept at rest.]

84. (*a*) The graph is shown. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.84a."



(*b*) Equilibrium occurs at the location where the force is 0. Set the force equal to 0 and solve for the separation distance *r*.

$$F(r_0) = -\frac{C}{r_0^2} + \frac{D}{r_0^3} = 0 \quad \rightarrow$$

$$\frac{C}{r_0^2} = \frac{D}{r_0^3} \quad \rightarrow \quad Cr_0^3 = Dr_0^2 \quad \rightarrow \quad \boxed{r_0 = \frac{D}{C}}$$

This does match with the graph, which shows $F = 0$ at $r = D/C$.

(*c*) We find the net force at $r = r_0 + \Delta r$. Use the binomial expansion.

$$F\left(r_0 + \Delta r\right) = -C\left(r_0 + \Delta r\right)^{-2} + D\left(r_0 + \Delta r\right)^{-3} = -Cr_0^{-2}\left(1 + \frac{\Delta r}{r_0}\right)^{-2} + Dr_0^{-3}\left(1 + \frac{\Delta r}{r_0}\right)^{-3}$$

$$\approx -\frac{C}{r_0^2}\left(1 - 2\frac{\Delta r}{r_0}\right) + \frac{D}{r_0^3}\left(1 - 3\frac{\Delta r}{r_0}\right) = \frac{C}{r_0^3}\left[-r_0\left(1 - 2\frac{\Delta r}{r_0}\right) + \frac{D}{C}\left(1 - 3\frac{\Delta r}{r_0}\right)\right]$$

$$= \frac{C}{r_0^3}\left[-r_0 + 2\Delta r + r_0 - 3\Delta r\right] = \frac{C}{r_0^3}\left[-\Delta r\right] \rightarrow F\left(r_0 + \Delta r\right) = -\frac{C}{r_0^3}\Delta r$$

We see that the net force is proportional to the displacement and in the opposite direction to the displacement. Thus the motion is simple harmonic.

(*d*) Since for simple harmonic motion, the general form is $F = -kx$, we see that for this situation, the spring constant is given by $k = \frac{C}{r_0^3} = \boxed{\frac{C^4}{D^3}}$.

(*e*) The period of the motion can be found from Eq. 14-7b.

$$T = 2\pi\sqrt{\frac{m}{k}} = \boxed{2\pi\sqrt{\frac{mD^3}{C^4}}}$$

86. Carbon dioxide is a linear molecule. The carbon–oxygen bonds in this molecule act very much like springs. Figure 14–43 shows one possible way the oxygen atoms in this molecule can oscillate: the oxygen atoms oscillate symmetrically in and out, while the central carbon atom remains at rest. Hence each oxygen atom acts like a simple harmonic oscillator with a mass equal to the mass of an oxygen atom. It is observed that this oscillation occurs with a frequency of $f = 2.83 \times 10^{13}$ Hz. What is the spring constant of the C—O bond?

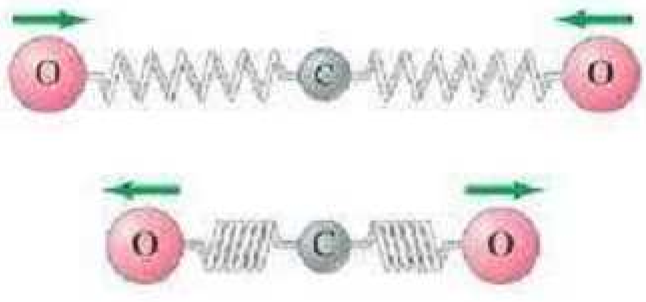


FIGURE 14–43 Problem 86, the CO_2 molecule.

86. The effective spring constant is determined by the frequency of vibration and the mass of the oscillator. Use Eq. 14-7a.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow$$

$$k = 4\pi^2 f^2 m = 4\pi^2\left(2.83\times10^{13}\,\text{Hz}\right)(16.00\,\text{u})\left(\frac{1.66\times10^{-27}\,\text{kg}}{1\,\text{u}}\right) = \boxed{840\,\text{N/m}} \text{ (3 sig. fig.)}$$